EMPIRICAL P–L–C RELATIONS FOR DELTA SCUTI STARS

S. K. GUPTA
Uttar Pradesh State Observatory, Naini Tal, India

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Abstract. Separate P–L–C relations have been empirically derived by sampling the δ Scuti stars according to their pulsation modes. The results based on these relations have been compared with those estimated from the model based P–L–C relations and the other existing empirical P–L–C relations. It is found that a separate P–L–C relation for each pulsation mode provides a better correspondence with observations.

1. Introduction

Empirical P–L–C relations for δ Scuti variables, based on observational data, have been derived by different authors (Elliott, 1974; Breger and Bregman, 1975). While these relations satisfactorily apply to δ Scuti stars pulsating in the fundamental mode, they do not hold very well for δ Scuti stars pulsating in overtones. Based on available observational data (Petersen and Jorgenson, 1972; Baglin *et al.*, 1973; Breger and Bregman, 1975), our main aim in this paper is to obtain empirical P–L–C relations for δ Scuti stars pulsating in different pulsation modes. We have compared the results obtained from our P–L–C relations with those estimated from the model based P–L–C relations by Tsvetkov (1977) and with empirical P–L–C relations derived by Elliott (1974) and Breger and Bregman (1975) and with the photometrically derived values. We find that the estimates based on our formulations are, overall, statistically closer to the photometrically derived values as well as to Tsvetkov's (1977) estimates.

2. Period–Luminosity–Colour Relation

Elliott (1974) derived an empirical P–L–C relation for the δ Scuti stars in the form

$$\underset{\pm 0^{m}.26}{M_v} = -2.21 \log P + 5.44\,(b-y) - 1.68. \tag{1}$$

Subsequently, Breger and Bregman (1975) derived empirical P–L–C relations in the following form

$$\underset{\pm 0^{m}.22}{M_v} = -2.74 \log P_0 + 3.8\,(b-y) - 1.94, \tag{2}$$

$$\underset{\pm 0^{m}.23}{M_{\mathrm{bol}}} = -2.82 \log P_0 - 7.6 \log T_e + 27.8. \tag{3}$$

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Recently, based on linear non-adiabatic model calculations, Tsvetkov (1977) has derived the following P–L–C relations for δ Scuti stars pulsating in the fundamental (F), first overtone (1H) and second overtone (2H) pulsation modes:

$$\left.\begin{array}{lll} [\mathrm{F}] & M_{\mathrm{bol}} = -3.87 \log P_0 - 11.99 \log T_e + 44.19, \\ & \pm 0\overset{\mathrm{m}}{.}03 \quad \pm 0.03 \qquad \pm 0.15 \qquad \pm 0.55 \\ [1\mathrm{H}] & M_{\mathrm{bol}} = -3.91 \log P_1 - 12.02 \log T_e + 43.81, \\ & \pm 0\overset{\mathrm{m}}{.}02 \quad \pm 0.02 \qquad \pm 0.11 \qquad \pm 0.42 \\ [2\mathrm{H}] & M_{\mathrm{bol}} = -3.94 \log P_2 - 12.00 \log T_e + 43.33. \\ & \pm 0\overset{\mathrm{m}}{.}02 \quad \pm 0.02 \qquad \pm 0.10 \qquad \pm 0.36 \end{array}\right\} \quad (4)$$

Taking our cue from Tsvetkov's work, we sampled the δ Scuti variables according to their pulsation modes and have derived the following least-squares solutions of P–L–C relations for each mode of pulsation

$$\left.\begin{array}{ll} [\mathrm{F}] & M_v = -2.73 \log P_0 + 4.17\,(b-y) - 1.97, \\ & \pm 0.16 \\ [1\mathrm{H}] & M_v = -2.38 \log P_1 + 4.34\,(b-y) - 1.93, \\ & \pm 0\overset{\mathrm{m}}{.}18 \\ [2\mathrm{H}] & M_v = -2.18 \log P_2 + 5.59\,(b-y) - 1.90, \\ & \pm 0\overset{\mathrm{m}}{.}21 \end{array}\right\} \quad (5)$$

$$\left.\begin{array}{ll} [\mathrm{F}] & M_{\mathrm{bol}} = -2.83 \log P_0 - 11.07 \log T_e + 41.61, \\ & \pm 0\overset{\mathrm{m}}{.}20 \\ [1\mathrm{H}] & M_{\mathrm{bol}} = -3.57 \log P_1 - 10.21 \log T_e + 37.13, \\ & \pm 0\overset{\mathrm{m}}{.}14 \\ [2\mathrm{H}] & M_{\mathrm{bol}} = -2.45 \log P_2 - 10.22 \log T_e + 38.35. \\ & \pm 0\overset{\mathrm{m}}{.}24 \end{array}\right\} \quad (6)$$

In deriving the relations (5) and (6) for the fundamental [F] mode, we have excluded the stars HR 3185 and δ Scuti itself because the **uvby** β-photometric absolute magnitudes of these two stars are less certain due to their large $[m_1]$ indices and also since the β-temperature and the $(b-y)$-temperature of these stars differ considerably (Leung 1970; Breger and Bregman, 1975).

In Table I(a), columns (2), (3) and (4), we tabulate the absolute magnitudes for 22 δ Scuti stars known to be oscillating in the fundamental mode, based respectively on relations (1), (2) and [F] of (5). In the fifth column of the same table are tabulated the absolute magnitudes as derived from **uvby** β-photometry (Baglin *et al.* 1973). Based on these, the deviations of the photometrically derived values from those computed on different formulations have been listed in the last three columns of the table. The r.m.s. values of these deviations have also been obtained.

Likewise, comparison between estimates based on relations (1), (2) and [1H] of (5) for 12 stars oscillating in the 1H mode is given in Table I(b) and the comparison between estimates based on relations (1), (2) and [2H] of (5) for 13 stars oscillating in the 2H mode is given in Table I(c).

TABLE I(a)

Absolute magnitudes of δ Scuti stars oscillating in the fundamental mode

Star HR	Elliott (E) M_v	Breger (B) M_v	Gupta (G) M_v	Photometry (P) M_v	E–P	B–P	G–P
515	0m92	0m82	0m84	0m78	0m14	0m04	0m06
729	1.59	1.65	1.67	1.99	−0.40	−0.34	−0.32
1170	1.55	1.54	1.57	1.53	0.02	0.01	0.04
1225	1.18	1.16	1.18	1.29	−0.11	−0.13	−0.11
1287	1.34	1.18	1.22	1.18	0.16	0.00	0.04
1351	2.05	2.18	2.20	2.38	−0.33	−0.20	−0.18
2107	1.07	1.02	1.04	0.94	0.13	0.08	0.10
3185	1.60	1.37	1.43	1.82	−0.22	−0.45	−0.39
3265	1.87	1.88	1.91	2.04	−0.17	−0.16	−0.13
HD 73450	1.99	2.17	2.18	2.35	−0.36	−0.18	−0.17
3888	1.33	1.21	1.25	1.06	0.27	0.15	0.19
5005	1.44	1.33	1.37	1.08	0.36	0.25	0.29
5017	1.22	1.13	1.15	1.16	0.06	−0.03	−0.01
5960	1.94	2.03	2.05	2.21	−0.27	−0.18	−0.16
6290	1.66	1.59	1.63	1.77	−0.11	−0.18	−0.14
6391	1.46	1.58	1.59	1.80	−0.34	−0.22	−0.21
7020	0.94	0.74	0.77	1.62	−0.68	−0.88	−0.85
7331	1.03	0.94	0.96	0.75	0.28	0.19	0.21
7501	1.86	1.80	1.84	1.79	0.07	0.01	0.05
7563	1.51	1.48	1.51	1.37	0.14	0.11	0.14
7928	1.16	1.02	1.05	1.22	−0.06	−0.20	−0.17
8006	1.83	1.91	1.93	1.90	−0.07	0.01	0.03
				r.m.s. values:	0.21	0.18	0.16

TABLE I(b)

Absolute magnitudes of δ Scuti stars oscillating in the first harmonic mode

Star HR	Elliott (E) M_v	Breger (B) M_v	Gupta (G) M_v	Photometry (P) M_v	E–P	B–P	G–P
21	1m67	1m57	1m35	1m20	0m47	0m37	0m15
432	0.76	0.82	0.56	0.52	0.24	0.30	0.04
1298	1.82	1.81	1.54	1.60	0.22	0.21	−0.06
2707	1.53	1.50	1.25	1.09	0.44	0.41	0.16
2989	1.11	1.23	0.99	1.03	0.08	0.20	−0.04
HD 73576	1.42	1.60	1.26	1.42	0.00	0.18	−0.16
HD 73729	1.90	1.91	1.62	2.04	−0.14	−0.13	−0.42
HD 74028	1.79	2.01	1.63	1.81	−0.02	0.20	−0.18
HD 74050	1.68	1.89	1.51	1.53	0.15	0.36	−0.02
4684	1.64	1.89	1.50	1.71	−0.07	0.18	−0.21
4715	1.23	1.01	0.86	0.71	0.52	0.30	0.15
7222	1.58	1.48	1.26	1.10	0.48	0.38	0.16
				r.m.s. values:	0.30	0.28	0.18

TABLE I(c)

Absolute magnitudes of δ Scuti stars oscillating in the second harmonic mode

Star HR	Elliott (E) M_v	Breger (B) M_v	Gupta (G) M_v	Photometry (P) M_v	E–P	B–P	G–P
114	1.80	1.87	1.57	1.46	0.34	0.41	0.11
401	2.30	2.63	2.05	2.17	0.13	0.46	0.12
812	2.22	2.50	1.97	2.25	−0.03	0.25	−0.28
1356	2.20	2.49	1.95	2.14	0.06	0.35	−0.19
1547	2.03	2.29	1.78	1.89	0.14	0.40	−0.11
1706	1.37	1.45	1.13	0.94	0.43	0.51	0.19
2100	1.63	1.83	1.40	1.34	0.29	0.49	0.06
HD 73175	2.17	2.45	1.93	2.11	0.06	0.34	−0.18
HD 73763	2.19	2.48	2.06	1.65	0.54	0.83	0.41
5329	1.55	1.73	1.31	0.98	0.57	0.75	0.33
8494	2.28	2.47	2.04	2.06	0.22	0.41	−0.02
8584	1.73	1.94	1.49	1.39	0.34	0.55	0.10
8880	1.63	1.86	1.39	1.02	0.61	0.74	0.27
				r.m.s. values:	0.32	0.52	0.21

TABLE II(a)

Bolometric magnitudes of δ Scuti stars oscillating in the fundamental mode

Star HR	Breger (B) M_{bol}	Tsvetkov (T) M_{bol}	Gupta (G) M_{bol}	Photometry (P) $M_{bol} = M_v$ + b.c.	B–P	T–P	G–P
242	1.24	1.73	1.68	1.60	−0.36	0.13	0.08
515	0.53	0.71	0.88	0.69	−0.16	0.02	0.19
729	1.36	1.84	1.69	1.92	−0.56	−0.08	−0.23
HD 23607	1.67	2.21	1.87	2.28	−0.61	−0.07	−0.41
HD 24550	1.51	2.06	1.88	1.75	−0.24	0.31	0.13
1170	1.28	1.75	1.67	1.47	−0.19	0.28	0.20
1225	0.90	1.22	1.26	1.23	−0.33	−0.01	0.03
1287	0.72	0.97	1.09	1.15	−0.43	−0.18	−0.06
1351	1.88	2.56	2.24	2.32	−0.44	0.24	−0.08
2107	0.84	1.15	1.23	0.91	−0.07	0.24	0.32
3185	0.93	1.31	1.38	1.82	−0.89	−0.51	−0.44
3265	1.56	2.14	1.95	2.02	−0.46	0.12	−0.07
HD 73450	1.86	2.53	2.18	2.28	−0.42	0.25	−0.10
3888	0.92	1.27	1.33	1.00	−0.08	0.27	0.33
5005	1.07	1.49	1.51	1.03	0.04	0.46	0.48
5017	0.76	1.02	1.11	1.14	−0.38	−0.12	−0.03
5960	1.69	2.31	2.05	2.15	−0.46	0.16	−0.10
6290	1.35	1.88	1.81	1.74	−0.39	0.14	0.07
6391	1.28	1.70	1.55	1.72	−0.44	−0.02	−0.17
7020	0.40	0.55	0.79	1.60	−1.20	−1.05	−0.81
7331	0.59	0.78	0.92	0.69	−0.10	0.09	0.23
7501	1.46	2.03	1.90	1.73	−0.27	0.30	0.17
7563	1.17	1.60	1.54	1.31	−0.14	0.29	0.23
7928	0.74	1.02	1.15	1.16	−0.42	−0.14	−0.01
8006	1.60	2.18	1.95	1.84	−0.24	0.34	0.11
				r.m.s. values:	0.38	0.22	0.20

TABLE II(b)

Bolometric magnitudes of δ Scuti stars oscillating in the first harmonic mode

Star HR	Breger (B) M_{bol}	Tsvetkov (T) M_{bol}	Gupta (G) M_{bol}	Photometry (P) $M_{bol} = M_v$ + b.c.	B–P	T–P	G–P
21	1m30	1m36	1m32	1m17	0m13	0m19	0m15
431	1.48	1.61	1.54	1.60	−0.12	0.01	−0.06
432	0.54	0.22	0.32	0.34	0.20	−0.12	−0.02
1298	1.53	1.66	1.60	1.57	−0.04	0.09	0.03
2707	1.21	1.21	1.20	1.01	0.21	0.20	0.19
2989	0.97	0.82	0.87	1.00	−0.03	−0.18	−0.13
HD 73576	1.33	1.32	1.33	1.33	0.00	−0.01	0.00
HD 73729	1.63	1.79	1.72	2.01	−0.38	−0.22	−0.29
HD 74028	1.69	1.82	1.78	1.74	−0.05	0.08	0.04
HD 74050	1.58	1.66	1.64	1.46	0.12	0.20	0.18
4684	1.57	1.63	1.61	1.62	−0.05	0.01	−0.01
4715	0.73	0.57	0.59	0.69	0.04	−0.12	−0.10
7222	0.98	0.86	0.89	1.07	−0.09	−0.21	−0.18
				r.m.s. values:	0.15	0.15	0.14

TABLE II(c)

Bolometric magnitudes of δ Scuti stars oscillating in the second harmonic mode

Star HR	Breger (B) M_{bol}	Tsvetkov (T) M_{bol}	Gupta (G) M_{bol}	Photometry (P) $M_{bol} = M_v$ + b.c.	B–P	T–P	G–P
401	2m35	2m38	2m12	2m11	0m24	0m27	0m01
812	2.17	2.13	1.97	2.16	0.01	−0.03	−0.19
HD 23156	2.54	2.62	2.22	2.23	0.31	0.39	−0.01
HD 23567	2.44	2.51	2.24	2.23	0.21	0.28	0.01
HD 23643	2.08	1.94	1.73	1.79	0.29	0.15	−0.06
1356	2.17	2.12	1.94	2.07	0.10	0.05	−0.13
1547	1.98	1.85	1.78	1.77	0.21	0.08	0.01
1706	1.16	0.73	1.11	0.87	0.29	−0.14	0.24
2100	1.56	1.28	1.43	1.22	0.34	0.06	0.21
HD 73175	2.18	2.15	2.00	2.05	0.13	0.10	−0.05
HD 73763	2.22	2.20	2.02	1.58	0.64	0.62	0.44
5329	1.47	1.14	1.35	0.90	0.57	0.24	0.45
8494	2.23	2.25	2.12	2.00	0.23	0.25	0.12
8584	1.63	1.36	1.47	1.31	0.32	0.05	0.16
8880	1.60	1.33	1.45	0.90	0.70	0.43	0.55
				r.m.s. values:	0.36	0.26	0.24

Corresponding results for the bolometric magnitudes are given in Tables II(a), II(b) and II(c). In these the estimates of absolute bolometric magnitudes based on formulations (2), (4) and (6) have been compared against those obtained photometrically from the relation

$$M_{\text{bol}} = M_v + \text{b.c.},$$

where the bolometric corrections (Allen, 1973), applicable for Main Sequence stars, have been used for δ Scuti stars, as the δ Scuti stars are assumed to be in the post-Main Sequence phase of evolution.

3. Discussions

It is seen from Table I(a) that for δ Scuti stars pulsating in the fundamental mode, the r.m.s. deviations in the various cases do not differ significantly, whereas from Tables I(b) and I(c) it is apparent that, for the first and second overtone pulsators, the r.m.s. deviations for our estimates are significantly lower. Likewise, the r.m.s. deviations in the case of M_{bol} (Tables II(a), II(b) and II(c)), based on our formulations, are statistically similar to the values based on Tsvetkov's formulations for all the pulsation modes, while Breger and Bregman's value agrees for the first overtone pulsators only. The absolute magnitudes, computed from Elliott's P–L–C relation are, on average, $0^{\text{m}}3$ higher than the photometric M_v values for the first and second overtone pulsators. Likewise, those computed from Breger and Bregman's P–L–C relation are, respectively, $0^{\text{m}}3$ and $0^{\text{m}}5$ higher than the photometric M_v values (Tables I(b) and I(c)) for the first and second overtone pulsators. Thus, overall, it is found that rather than a common P–L–C relation holding good for all δ Scuti stars, it is more meaningful to think in terms of separate P–L–C relations for δ Scuti stars pulsating in different pulsation modes.

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