THE EFFECT OF METALLICITY ON THE EMPIRICAL RELATIONS FOR RR LYRAE STARS

I: *Effective Temperature and Colour*

H. S. MAHRA and S. D. SINVHAL*
Uttar Pradesh State Observatory, Manora Peak, Naini Tal, India

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Abstract. It has empirically been shown that, for a given value of the effective temperature, the correction in $(B - V)$ due to line blanketing varies linearly with the metallicity parameter ΔS. Next, on the basis of considerations different from those used to obtain a similar result by Sturch (1966), a relation between reciprocal effective temperature θ_e $(=5040/T_e)$ and intrinsic line-free colour index $(B - V)_{o,c}$ has been obtained based on the θ_e and $(B - V)$ values for five RR Lyr variables. Relations between θ_e and intrinsic colour $(B - V)_o$ for different groups of stars having a ΔS parameter in the range $0 \leqslant \Delta S \leqslant 11$ have also been obtained.

1. Introduction

The effective temperatures of most RR Lyr variables are usually obtained, among other methods, through the use of empirical relations between the reciprocal effective temperature θ_e and either the intrinsic reddening-free photometric colour $(B - V)_o$ or the intrinsic reddening-cum-blanketing free colour $(B - V)_{o,c}$ (Oke *et al.*, 1962; Searle and Oke, 1962; Preston and Paczynski, 1964; Oke, 1966; Sturch, 1966). This is a powerful, and perhaps the only, tool for obtaining the temperatures of faint RR Lyr variables from photometric observations. However, the various empirical relationships given by different authors for obtaining the effective temperatures of RR Lyr variables, yield estimates differing by as much as 600 K. In the following sections we first examine these empirical relations and their underlying assumptions and conclude that the inaccuracies arise because the available formulations do not take into account the effect of metallicity. Subsequently, we give formulations which take this factor into account.

2. Relationship between Line-Blanketing Correction $\Delta(B - V)$ and the Metallicity Parameter ΔS

From studies of spectrograms taken with dispersions of 10–20 Å mm^{-1}, the line-blanketing corrections $\Delta(B - V)$ $[\equiv(B - V) - (B - V)_c]$ – where $(B - V)$ and $(B - V)_c$ are, respectively, the apparent and the blanketing free colours – have been

* Present address: Kumaun University, Naini Tal-263001, India.

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TABLE I

The metallicity parameters ΔS and line-blanketing corrections $\Delta(B - V)$ for five RR Lyr stars given by earlier authors and the respective values obtained in this work

(1) Star	(2) ΔS	(3) $\Delta(B - V)$ (based on spectroscopic observations by earlier authors)	(4) $\Delta(B - V)$ (this work)	(5) Difference (3) − (4)
SW And	0	$0^m.11$	$0^m.11$	$0^m.00$
DX Del	2	0.10	0.09	+0.01
RR Lyr	6	0.04	0.05	−0.01
SU Dra	8	0.04	0.03	+0.01
X Ari	10	0.01	0.01	0.00

obtained for the five RR Lyr stars SW And, DX Del, RR Lyr, SU Dra and X Ari, at phases corresponding to their near maximum θ_e values by Preston (1961), Oke *et al.* (1962) and Sturch (1966). These, together with the ΔS values given by Preston (1959), are tabulated in the first three columns of Table I and are plotted in Figure 1. Preston (1959) has obtained $\Delta S = 10$ for X Ari and SU Dra. Later, Oke (1966) showed that the metal to hydrogen ratio (m/H) for X Ari is one-fifth of that for SU

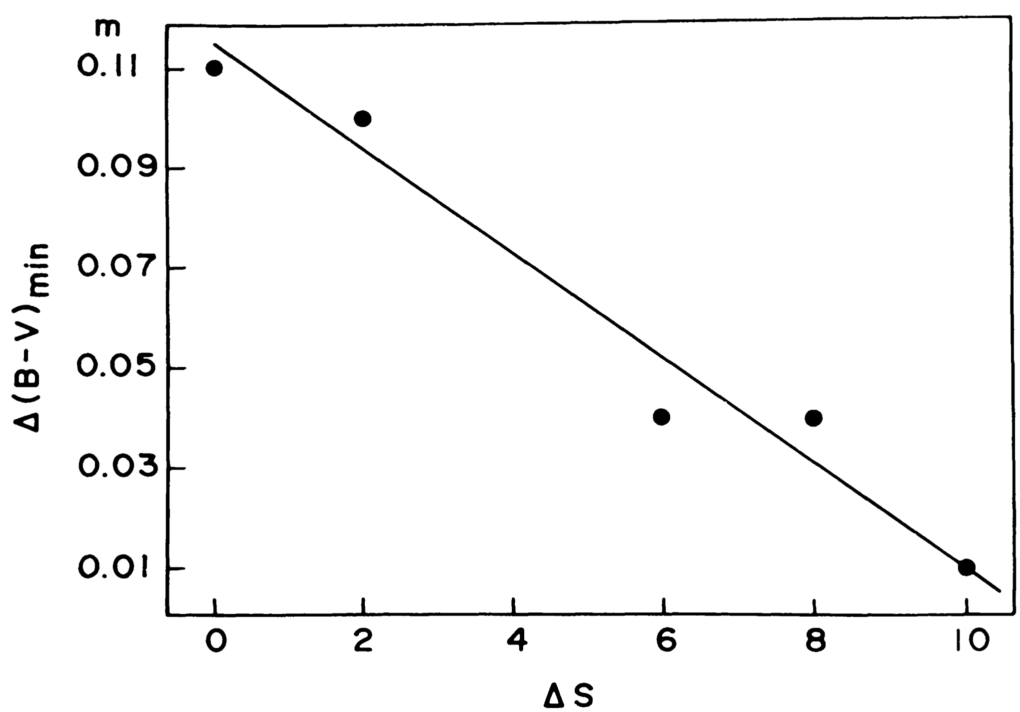

Fig. 1. Plot of $\Delta(B - V)_{min}$ against parameter ΔS.

Dra. Using the relation (Kraft, 1972), $\log [(m/\mathrm{H})\,\mathrm{star}/(m/\mathrm{H})_\odot] = 0.44 - 0.29\,\Delta S$; $\Delta S \geqslant 2$ we estimate the difference in ΔS values of the two stars to be about 2. Therefore, a value of $\Delta S = 8$ for SU Dra is more appropriate.

Preston (1959) has found that, for a great majority of RRab stars, the hydrogen type spectral class near minimum light is F5, with a dispersion amounting to one-tenth of a spectral class. Further, Sturch (1966) has shown that the $(B - V)$ colours of RRab stars during the phase intervals $0^{\mathrm{p}}5 < \phi < 0^{\mathrm{p}}8$ can be treated as constant. During the phase interval $0^{\mathrm{p}}4 < \phi < 0^{\mathrm{p}}85$, the θ_e values for SU Dra range from 0.83 to 0.84 (Oke *et al.*, 1962) and those for X Ari range from 0.82 to 0.84 (Oke, 1966), the variation in the θ_e values being within the accuracy of θ_e determination. One can, therefore, reasonably assume that the line-blanketing corrections given in Table I are for a common value of $\theta_e = 0.83$. These are plotted in Figure 1 whence we infer that, at least for this particular value of θ_e, the blanketing correction $\Delta(B - V)$ varies linearly with ΔS. This partly justifies the tacit general assumption made by Dickens (1970) about the linear relationship between $\Delta(B - V)$ and ΔS, which we too shall adopt.

3. The Line-Blanketing Correction $\Delta(B - V)$ for RRab Stars

The effect of metallic line-blanketing on the U, B, V magnitudes of normal Main Sequence stars has been investigated by Sandage and Eggen (1959), Melbourn (1960), Wildey *et al.* (1962) and Mihalas (1966). Based on these results, Dickens (1970) has obtained the values of $\Delta(B - V)$ for different values of θ_e for RR Lyr stars having a normal metal to hydrogen ratio corresponding to $\Delta S = 0$. These values are given in the second column of Table II, in which the third column gives the corresponding values on the assumption of a linear relationship. The above data show that it is reasonable to assume that for the case $\Delta S = 0$, the variation of $\Delta(B - V)$ with θ_e is linear. Coupled with the assumption that for a given value of θ_e, the excess $\Delta(B - V)$ varies linearly also with ΔS, and that $\Delta(B - V) = 0$ for an extrapolated value of $\Delta S = 11$, we obtain the expression

$$\Delta(B - V) = (0.32\theta_e - 0.16)(1 - 0.09\,\Delta S); \qquad 0.60 \leqslant \theta_e \leqslant 0.80. \qquad (1)$$

TABLE II

Variation of blanketing correction with temperature, for stars with $\Delta S = 0$

θ_e	0.60	0.65	0.70	0.75	0.80
$\Delta(B - V)$ (Dickens, 1970)	$0^{\mathrm{m}}03$	0.05	0.06	0.07	0.10
$\Delta(B - V)$ (linear relation)	$0^{\mathrm{m}}03$	0.05	0.06	0.08	0.10

On the basis of relation (1), we have calculated the values of $\Delta(B - V)$ for the five RR Lyr stars SW And, DX Del, RR Lyr, SU Dra and X Ari, corresponding to a common value of $\theta_e = 0.83$. These calculated values of $\Delta(B - V)$, given in column (4) of Table I, are in good agreement with the earlier values obtained from the studies of high-dispersion spectrograms of these stars. In subsequent discussions we assume that relation (1) can be extended to $\theta_e = 0.85$.

4. The Relation between θ_e and $(B - V)_{o,c}$

From the plots of θ_e and $(B - V)$, Preston and Paczynski (1964) inferred that the straight lines fitted to the data of the five individual stars have 'similar' slopes with a mean value of 0.63 and that these have different intercepts.

Since the intrinsic colours of the stars are affected by line blanketing, which in turn depends upon the ΔS parameter and the effective temperature of the star, the estimation of θ_e on the basis of a mean value of the slope of θ_e, $(B - V)$ lines, as has been done by Preston and Paczynski (1964), does not appear justified.

Based on relation (1), the line-blanketing corrections $\Delta(B - V)$ corresponding to the observed θ_e and ΔS parameters of these five stars have been estimated, and thence the blanketing free colours $(B - V)_c$ have been obtained. The data are tabulated in columns (1)–(6) of Table III and the $(B - V)_c$ values are plotted against the corresponding θ_e values in Figure 2, in which the almost parallel shifting of the lines with respect to one another should now be due to the different amounts of interstellar reddening suffered by the individual stars. We have measured the shift of these lines with respect to the star RR Cet which has been assumed to be unreddened. The mean values of the reddening corrections $E(B - V)$ thus obtained for these stars are given in column (7) of Table III.

The reddening correction of $0\overset{m}{.}04$ obtained by us for TU UMa ($l = 198\overset{\circ}{.}77$, $b = +71\overset{\circ}{.}86$) is in good agreement with Sturch's (1966) determination of a general reddening of $0\overset{m}{.}03$ near the galactic pole. Likewise, Preston and Paczynski (1964) and Oke (1966) have obtained a reddening correction of $0\overset{m}{.}19$ for X Ari, which is in good agreement with our derived value of $0\overset{m}{.}21$. Therefore, our assumption that the star RR Cet is unreddened is justified and we have adopted the values of $E(B - V)$ $[\equiv (B - V) - (B - V)_o]$ as given in Table III.

After corrections for reddening and line blanketing have been applied to the observed $(B - V)$ values, the intrinsic line-free colour indices $(B - V)_{o,c}$ for the five stars have been obtained and tabulated in column (8) of Table III. A plot of these $(B - V)_{o,c}$ values against the corresponding θ_e values in Figure 3 yields a least-squares linear fit, given by the relation

$$\theta_e = 0.69(B - V)_{o,c} + 0.61. \quad (2)$$

In comparing the above relation with a similar relation $\theta_e = 0.74(B - V)_{o,c} + 0.61$ obtained by Sturch (1966), we note that (a) the latter has neglected the correction

TABLE III

The intrinsic line-free colour index and θ_e value for five RR Lyr stars given by earlier authors and the respective values obtained in this work

(1) Star	(2) ΔS	(3) $\theta_e(H\gamma)$ (earlier authors)	(4) $(B-V)$	(5) $\Delta(B-V)$	(6) $(B-V)_c$	(7) $E(B-V)$	(8) $(B-V)_{o,c}$	(9) $(B-V)_o$	(10) θ_e (this work)	(11) $\Delta\theta_e$ (3) − (10)
SW And	0	0.66	$0^m.20$	$0^m.05$	$0^m.15$	$0^m.05$	$0^m.10$	0 15	0.68	−0.02
		0.69	0.20	0.06	0.14		0.09	0.15	0.68	+0.01
		0.85	0.51	0.11	0.40		0.35	0.46	0.85	0.00
RR Cet	5	0.69	0.16	0.03	0.13	0.00	0.13	0.16	0.70	−0.01
		0.71	0.16	0.04	0.12		0.12	0.16	0.70	+0.01
		0.82	0.35	0.06	0.29		0.29	0.35	0.82	0.00
		0.83	0.41	0.06	0.35		0.35	0.41	0.85	−0.02
TU UMa	6	0.68	0.17	0.03	0.14	0.04	0.10	0.13	0.68	0.00
		0.84	0.43	0.05	0.38		0.34	0.39	0.85	−0.01
RX Eri	9	0.68	0.24	0.01	0.23	0.12	0.11	0.12	0.69	−0.01
		0.84	0.47	0.02	0.45		0.33	0.35	0.84	0.00
		0.84	0.48	0.02	0.46		0.34	0.36	0.85	−0.01
X Ari	10	0.66	0.29	0.00	0.29	0.21	0.08	0.08	0.66	0.00
		0.66	0.30	0.00	0.30		0.09	0.09	0.66	0.00
		0.83	0.53	0.01	0.52		0.31	0.32	0.83	0.00
		0.83	0.55	0.01	0.54		0.33	0.34	0.84	−0.01
		0.84	0.54	0.01	0.53		0.32	0.33	0.83	+0.01

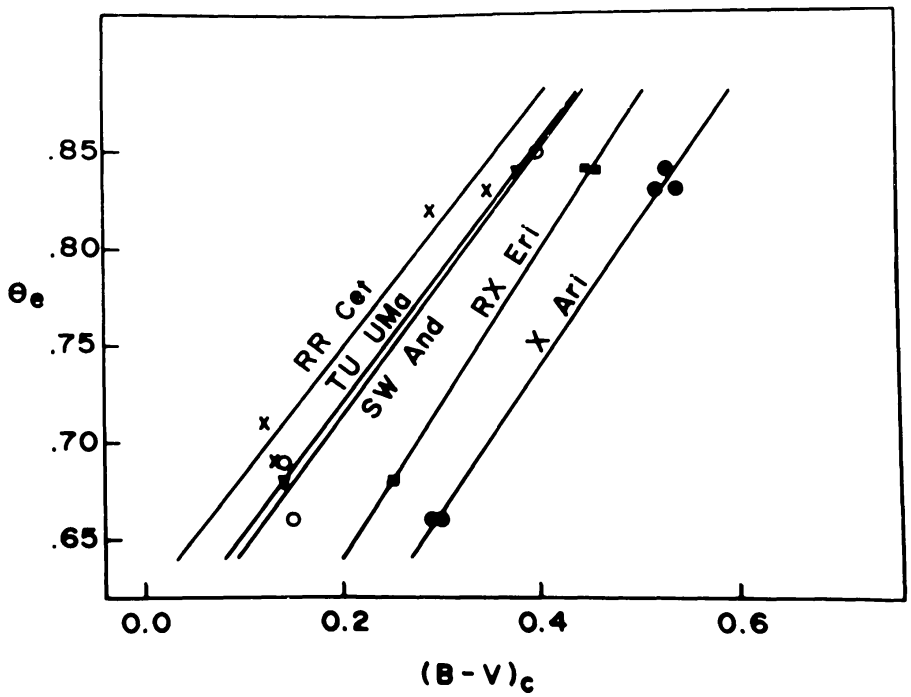

Fig. 2. Straight lines fitted to θ_e, $(B - V)_c$ data of individual stars.

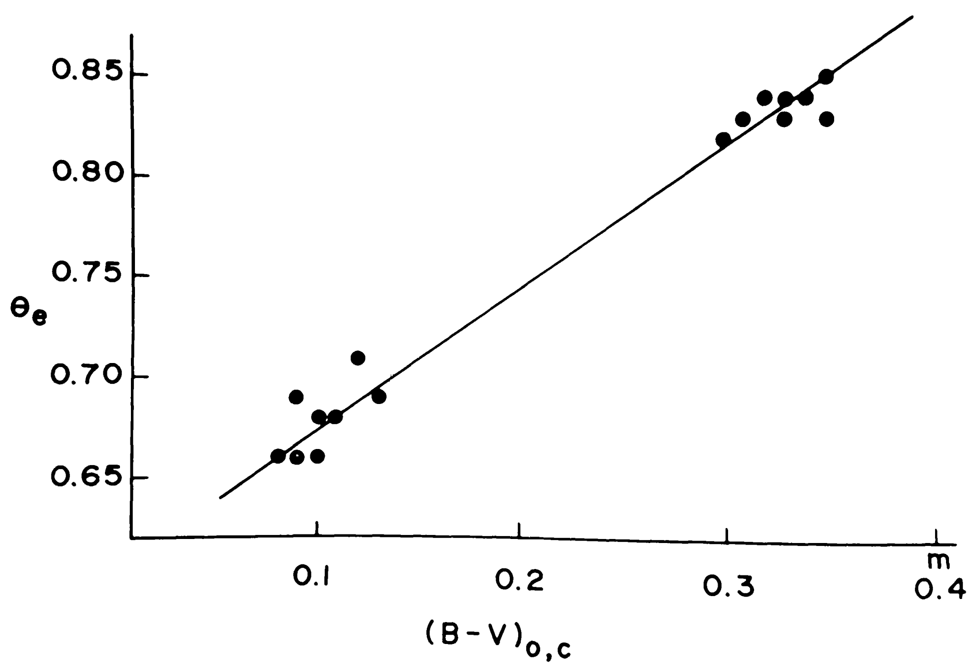

Fig. 3. Plot of $(B - V)_{o,c}$ values around maximum and minimum of the $(B - V)$ colour curves against the corresponding θ_e values for five RRa stars. The straight line is a least-squares fit.

$\Delta(B - V)$ corresponding to minimum θ_e values; (b) the values of $(B - V)_{o,c}$, corresponding to maximum θ_e values, have been obtained by him from an empirical relation between period and intrinsic line-free colour index; and (c) it appears that the corrections $\Delta(B - V)$ corresponding to minimum θ_e values, which have been neglected by Sturch (1966) in obtaining the maximum $(B - V)_{o,c}$, are compensated for because he has applied larger corrections for interstellar reddening.

5. The Relation between Intrinsic Colour Index $(B - V)_o$ and θ_e

Substituting relation (1) in (2) and recalling that $\Delta(B - V) \equiv (B - V) - (B - V)_c \equiv (B - V)_o - (B - V)_{o,c}$ we obtain:

$$\theta_e = 0.69(B - V)_o - (0.22\theta_e - 0.11)(1 - 0.09\,\Delta S) + 0.61\,. \quad (3)$$

Substituting the values of ΔS (= 0, 2, 4, ..., 10, 11) in the above, we obtained and tabulated in column (2) of Table IV the relationships between θ_e and $(B - V)_o$ for different groups of stars having a differing ΔS parameter.

5.1. Discussion

We now discuss the formulations given by us in Table IV, and compare them with those given by earlier authors:

ΔS = *11:* This has already been discussed in Section 4.

ΔS = *10:* The slope in the relation obtained by Oke (1966) for the star X Ari (ΔS = 10) is almost identical to the slope in the relation obtained by us for stars having a parameter ΔS = 10, although the intercepts in both relations differ by 0.05. The $(B - V)$ values for X Ari, used by Oke, are systematically redder by nearly $0^m.04$ than the values of $(B - V)$ for X Ari given by Preston and Paczynski (1964). Further, the reddening correction for X Ari applied by Oke (1966) is less by $0^m.02$ than

TABLE IV

The relation between θ_e and $(B - V)_o$

ΔS	Relation between θ_e and $(B - V)_o$	
	This work	Given by earlier authors
0	$\theta_e = 0.57(B - V)_o + 0.59$	—
2	$= 0.58(B - V)_o + 0.59$	—
4	$= 0.60(B - V)_o + 0.60$	—
6	$= 0.63(B - V)_o + 0.60$	$\theta_e = 0.63(B - V)_o + 0.59$ (Preston and Paczynski, 1964)
8	$= 0.65(B - V)_o + 0.60$	$= 0.51(B - V)_o + 0.62$ (Oke *et al.*, 1962)
10	$= 0.68(B - V)_o + 0.61$	$= 0.69(B - V)_o + 0.56$ (Oke 1966)
11	$= 0.69(B - V)_o + 0.61$	$= 0.74(B - V)_o + 0.61$ (Sturch, 1966)

the value of reddening correction obtained by us. Thus, the $(B - V)_0$ values for X Ari used by Oke (1966), for obtaining the relation between θ_e and $(B - V)_0$, are systematically redder by nearly $0^m.06$. After applying a correction of $-0^m.06$ to the $(B - V)_0$ values of X Ari, the relation given by Oke (1966) becomes $\theta_e = 0.69(B - V)_0 + 0.60$, which is almost identical to the relation obtained by us for stars having a parameter $\Delta S = 10$.

$\Delta S = 8$: The value of the slope in the relation obtained by Oke *et al.* (1962) for SU Dra ($\Delta S = 8$) is quite different from the slope in the relation obtained by us for stars having $\Delta S = 8$. Oke (1966) has pointed out that the difference in line blanketing for X Ari and SU Dra does not fully explain the difference between the slopes of the two equations obtained for X Ari and SU Dra, and has mentioned the following possible causes to explain the difference:

(i) X Ari may have some kind of long period cycle similar to RR Lyr with consequent variations in $(B - V)$;
(ii) the colours of the two stars could have some systematic errors; and
(iii) the derived θ_e scale could be incorrect.

In so far as the first of these possibilities is concerned, the results obtained by Mahra and Sinvhal (1975) show that X Ari does not manifest the long period cycle similar to that of RR Lyr. The period of the star is increasing at a rate of 9.7×10^{-7} day per 1000 cycles and we have verified that this rate of change in the period of the star would not produce any appreciable change in the θ_e, $(B - V)_0$ relationship. Therefore, it appears that either the $(B - V)_0$ values of SU Dra used by Oke *et al.* (1962) may have some systematic error, or that the θ_e scale derived for SU Dra may be incorrect.

$\Delta S = 6$: Preston and Paczynski (1964) have obtained the relation between θ_e and $(B - V)_0$ on the basis of a mean value of the slopes of θ_e and $(B - V)_0$ for five RR Lyr stars for which the mean value of the ΔS parameter is found to be $\Delta S = 6$. From column (2) of Table IV it is seen that the slope and the intercept in the θ_e, $(B - V)_0$ relationships vary almost linearly with the ΔS parameter. Therefore, the relation given by Preston and Paczynski (1964) should be applicable to stars having parameter $\Delta S = 6$. If we compare this relationship with that obtained by us for stars having a parameter $\Delta S = 6$, we find that the two relations are almost identical.

Finally, based on relation (3) between θ_e, $(B - V)_0$ and ΔS and the intrinsic colour $(B - V)_0$ given in column (9) of Table III, we have calculated θ_e values for these five RR Lyr stars. These θ_e(cal) values are given in column (10) of Table III. The difference between the θ_e(cal) and θ_e(Hγ) obtained by Preston and Paczynski (1964) on the basis of H(γ) profiles is given in column (11) of Table III, whence it is apparent that the values of θ_e calculated on the basis of relation (3) are in good agreement with those obtained from H(γ) profiles.

References

Dickens, R. J.: 1970, *Astrophys. J. Suppl.* **22**, 249.
Kraft, R. P.: 1972, 'Evolution of Population II Stars', *Dudley Obs. Report*, No. 4, p. 69.

Mahra, H. S. and Sinvhal, S. D.: 1975, in 'Multiple Periodic Variable Stars', *IAU Coll.* **29**, 2039
Melbourn, W. G.: 1960, *Astrophys. J.* **132**, 101.
Mihalas, D.: 1966, *Astrophys. J. Suppl.* **13**, 1.
Oke, J. B.: 1966, *Astrophys. J.* **145**, 468.
Oke, J. B., Giver, L. P. and Searle, L.: 1962, *Astrophys. J.* **136**, 393.
Preston, G. W.: 1959, *Astrophys. J.* **130**, 507.
Preston, G. W.: 1961, *Astrophys. J.* **134**, 633.
Preston, G. W. and Paczynski, B.: 1964, *Astrophys. J.* **140**, 181.
Sandage, A. R. and Eggen, O. J.: 1959, *Monthly Notices Roy. Astron. Soc.* **119**, 278.
Searle, L. and Oke, J. B.: 1962, *Astrophys. J.* **135**, 790.
Sturch, C.: 1966, *Astrophys. J.* **143**, 774.
Wildey, R. L., Burbidge, E. M., Sandage, A. and Burbidge, G. R.: 1962, *Astrophys. J.* **135**, 94.