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THE EFFECT OF METALLICITY ON THE EMPIRICAL RELATIONS FOR RR LYRAE STARS

II: *The P–L–C Relations*

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Abstract. The coefficients in the *P–L–C* relation for RR Lyr stars show dependence on the metallicity parameter ΔS. For the range of metallicity and effective temperatures of RR Lyr stars, the effect of metallicity on the absolute magnitude ranges from $0^{m}.22$ to $0^{m}.42$. Modified *P–L–C* relations for RR Lyr stars have been derived.

1. A Critique of the Existing *P–L–C* Relations for RR Lyr Stars

Fernie (1964) was the first to deduce a *P–L–C* relation in the form

$$M_v = -2.50 \log P_0 + 2.96(B - V)_o - 0.83 \tag{1}$$

between the absolute magnitude M_v, the fundamental period P_0 and intrinsic colour $(B - V)_o$ of RR Lyr stars. He based his derivation on (i) a relation between θ_e and $(B - V)_0$ given by Oke *et al.* (1962) for only the one star SU Dra, which cannot be taken as a generalization for all RR Lyr variables because of the effect of metallicity on the relation between effective temperature and colour for RR Lyr stars (Mahra and Sinvhal, 1979, hereinafter called Paper I); and on (ii) an assumed period–radius relation $P_0 = k(R/R_\odot)^n$, where he assumed $n = 2$, essentially as a matter of simplification. Subsequently, Christy (1971) has, on theoretical considerations, obtained $n = 1.76$.

Later – based on observed periods, colours and apparent magnitude of RR Lyr stars in the globular clusters M3, ω Cen and M107 – Breger and Bregman (1975) deduced the *P–L–C* relations

$$M_v = -2.90 \log P_0 + 3.28(B - V)_o - 1.21 \pm 0.21, \tag{2}$$

where the upper sign in the last term applies to RRab stars and the lower sign to RRc stars. Breger and Bregman have also remarked that the coefficients in the *P–L–C* relations would be altered by a negligible amount due to any difference in the metal abundance between the different clusters – a remark which, we find, is not justified.

In view of these considerations, the above *P–L–C* relations need re-examination.

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2. Period–Radius Relation for RRab Stars

Starting with the relation between period, radius and mass given by Christy (1971), we obtain

$$\log (R/R_\odot) = 0.568 \log P_0 + 0.409 \log (M/M_\odot) + 0.953. \qquad (3)$$

If we consider the range of mass of RR Lyr variables to be small ($0.45 \leqslant M/M_\odot \leqslant 0.65$), relation (3) can be expressed as

$$\log (R/R_\odot) = 0.568 \log P_0 + \text{const.} \qquad (4)$$

In order to evaluate the constant term in the above relation, we have plotted in Figure 1 the $\log (R/R_\odot)$ against $\log P_0$ data for stars in the post-red-giant stage of evolution given by Fernie (1964, Table 3). In this we have omitted the star DQ Her for which the determination radius is uncertain. A least-squares linear solution provides an estimate of the constant term, and relation (4) can then be expressed as

$$\log (R/R_\odot) = 0.568 \log P_0 + 0.86. \qquad (5)$$

We adopt, after Fernie (1964), that a period–radius relation obtained from the data of these stars in the post-red-giant stage of evolution is applicable to RRab stars.

3. *P–L–C* Relation for RRab Stars

The relation between period, bolometric magnitude, M_{bol} and effective temperature, T_e, obtained by substituting the period–radius relation (5) in the fundamental relation $L/L_\odot = (R/R_\odot)^2(T_e/T_{e\odot})^4$, is found to be

$$M_{\text{bol}} = -2.84 \log P_0 - 10 \log T_e + 38.06, \qquad (6)$$

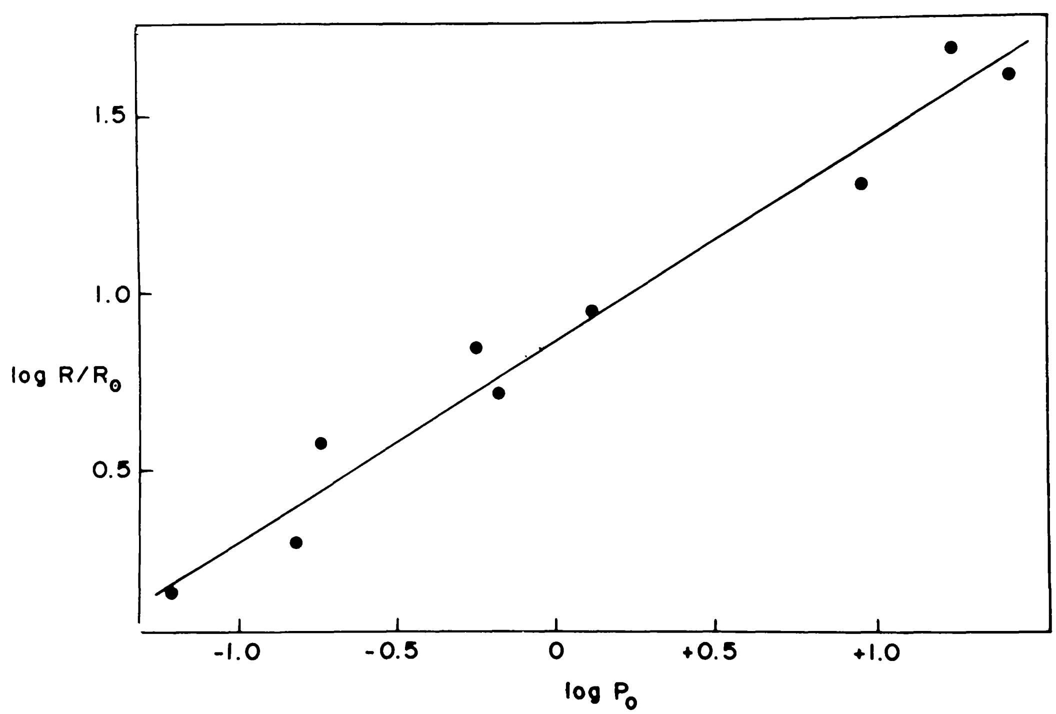

Fig. 1. The plot of $\log (R/R_\odot)$ against $\log P_0$ for stars in the post-red-giant stage of their evolution.

where the values $M_{\text{bol}\odot} = 4^{\text{m}}.75$ and $T_{e\odot} = 5770$ K have been adopted (Allen, 1973).

The relation between θ_e, $(B - V)_o$ and ΔS which we obtained in Paper I for RR Lyr stars having $0^{\text{m}}.1 \leqslant (B - V)_o \leqslant 0^{\text{m}}.6$ can be expressed as

$$\log T_e = -(0.317 + 0.0036\,\Delta S)(B - V)_o + (3.920 - 0.0016\,\Delta S). \tag{7}$$

Substituting relation (7) in relation (6), we obtain

$$M_{\text{bol}} = -2.84 \log P_0 + (3.17 + 0.036\Delta S)(B - V)_o - (1.14 - 0.016\Delta S). \tag{8}$$

Further, if we apply a mean bolometric correction of $0^{\text{m}}.02$ (Davis and Webb, 1970), relation (8) can be expressed as

$$M_v = -2.84 \log P_0 + (3.17 + 0.036\,\Delta S)(B - V)_o - (1.12 - 0.016\,\Delta S). \tag{9}$$

Thus, for a given value of P_0 and $(B - V)_o$, the difference in the absolute magnitude between the cases $\Delta S = 0$ and $\Delta S = 11$, obtained from relation (9), is found to be

$$\delta M_v = 0.40(B - V)_o + 0.18,$$

which, for the range of colour $0^{\text{m}}.1 \leqslant (B - V)_o \leqslant 0^{\text{m}}.6$ covered by RR Lyr stars, ranges from $0^{\text{m}}.22$ to $0^{\text{m}}.42$. Therefore, the remark by Breger and Bregman (1975) to the effect that the coefficients in the *P–L–C* relation would depend negligibly on metallicity, does not appear to be valid.

We mention, in passing, that the *P–L–C* relation (2) for RRab stars given by Breger and Bregman (1975), compares with the case $\Delta S = 3$, for which we obtain the relation $M_v = -2.84 \log P_0 + 3.28(B - V)_o - 1.07$ – the difference between the estimates in the two cases being approximately $0^{\text{m}}.1$.

4. *P–L–C* Relation for RRc Stars

Dickens (1970) estimated the metallicity parameter ΔS for stars in the globular cluster M3 to be between 3 and 6. Therefore, if we assume $\Delta S = 5$ for the stars in M3, the *P–L–C* relation applicable to RRab stars in M3 obtained from relation (8) is found to be

$$M_{\text{bol}} = -2.84 \log P_0 + 3.35(B - V)_o - 1.06. \tag{10}$$

Employing relation (10) and the photometric parameters of RR Lyr stars in M3 obtained by Sandage (1959), we formally estimated the bolometric magnitudes of all RR Lyr stars in M3, including RRc stars. The bolometric corrections corresponding to the $(B - V)_o$ values of the stars were obtained from the data of empirical bolometric corrections calculated by Davis and Webb (1970); and on this basis, the absolute magnitude M_v and distance modulus were estimated in each case. The mean values of estimates of the distance modulus of M3 based on thirty-four RRab stars and eleven

RRc stars, respectively, are $14^m\!.87 \pm 0^m\!.07$ (p.e.) and $14^m\!.49 \pm 0^m\!.06$ (p.e.), which differ by $0^m\!.38 \pm 0^m\!.09$ (p.e.). This implies that the bolometric magnitudes of RRc-type stars obtained from relation (10) are fainter by $0^m\!.38$ than those of RRab stars. Therefore, the *P–L–C* relation (8), which is basically valid for RRab stars alone (which pulsate in the fundamental mode with period P_0), will have to be correspondingly modified for RRc stars (which pulsate in the first overtone mode with period P_1). After applying this correction to relations (6) and (8), we obtain the following *P–L–C* relations for RRc stars:

$$M_{\text{bol}} = -2.84 \log P_1 - 10 \log T_e + 37.68 \tag{11}$$

$$= -2.84 \log P_1 + (3.17 + 0.036\,\Delta S)(B - V)_o - (1.52 - 0.016\,\Delta S) \tag{12}$$

and

$$M_v = -2.84 \log P_1 + (3.17 + 0.036\,\Delta S)(B - V)_o - (1.50 - 0.016\,\Delta S). \tag{13}$$

On the basis of above relationships, the *P–L–C* relation applicable to RRc stars having $\Delta S = 3$ is

$$M_v = -2.84 \log P_1 + 3.28(B - V)_o - 1.45,$$

and estimates based on this agree to within $0^m\!.1$ with those based on the *P–L–C* relation for RRc stars given by Breger and Bregman (1975).

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