SPECTROPHOTOMETRIC DETERMINATION OF THE TEMPERATURE OF THE CEPHEID SU CASSIOPEIAE

S. C. JOSHI and B. S. RAUTELA
Uttar Pradesh State Observatory, Naini Tal, India

(Received 26 October, 1979)

Abstract. The effective temperature of the short period cepheid SU Cas has been determined from a comparison of its spectral scans with appropriate model atmospheres. Using this temperature and an independent Wesselink radius determination, the luminosity of the star has been redetermined. The pulsation mass and the evolutionary mass of the star have been discussed. The evolutionary to pulsation mass ratio comes out to be greater than unity.

1. Introduction

In an earlier paper Rautela and Joshi (1976) have given the effective temperatures of ζ Geminorum at various phases of the light cycle, determined from a comparison of model atmosphere fluxes with the observed energy distribution curves of the star. Accurate determinations of the effective temperatures of cepheid variables are important in the sense that these, together with an independent knowledge of their radii, lead to better values of the luminosities of these stars. The luminosities so determined may improve the cepheid distance scale and also may help sort out the differences between the evolutionary and pulsation masses of cepheids. In this paper we have determined the effective temperatures of SU Cas at several phases using the scanner observations of the star. The radius, luminosity and mass of the star have also been derived. The ratio of evolutionary to pulsation mass is found to be greater than unity.

2. Observations

Photoelectric spectrophotometric scans of SU Cas were obtained on several nights during 1977–78 with the 104 cm reflector of the Uttar Pradesh State Observatory. The instrumental set up for these observations has been the same as described in our earlier paper (Rautela and Joshi, 1976). The bandwidth used for these measurements, however, was kept at 5 nm and a cooled ($-20°$ C) EMI 9658B photomultiplier has been used. The observations have been made at nineteen line-free wavelengths suggested by Oke (1965) in the wavelength interval 339–710 nm. The standard star γ Gem was also observed on each night to transform the extinction corrected instrumental magnitudes of the star to absolute values. These absolute values conform to the calibration of α Lyr as given by Hayes and Latham (1975).

Astrophysics and Space Science **70** (1980) 393–399. 0004–640X/80/0702–0393$01.05

TABLE I

Blanketing and reddening corrected magnitudes along with blanketing and reddening corrections for SU Cas at various phases (for each phase upper line gives blanketing and reddening corrected magnitudes while lower line gives blanketing corrections)

Phase \ $1/\lambda\ \mu m^{-1}$	2.95	2.90	2.85	2.75	2.70	2.48	2.40	2.35	2.24	2.19	2.09	2.00	1.90	1.80	1.71	1.65	1.55	1.47	1.41
0.072	5^{m}86	5^{m}83	5^{m}79	5^{m}86	5^{m}94	5^{m}10	5^{m}06	5^{m}06	5^{m}07	5^{m}05	5^{m}08	5^{m}08	5^{m}07	5^{m}09	5^{m}07	—	—	—	—
	0.37	0.39	0.42	0.42	0.42	0.20	0.26	0.24	0.19	0.21	0.10	0.13	0.16	0.07	0.04	—	—	—	—
0.189	5.79	5.76	5.67	5.64	5.75	5.12	5.11	5.12	5.10	5.09	5.11	5.11	5.14	5.15	5.14	5^{m}12	5^{m}11	5^{m}11	5^{m}12
	0.41	0.43	0.46	0.46	0.46	0.22	0.29	0.27	0.21	0.23	0.11	0.15	0.17	0.08	0.04	0.05	0.05	0.03	0.03
0.254	6.14	6.09	6.07	6.10	6.02	5.28	5.21	5.20	5.19	5.18	5.21	5.17	5.18	5.22	5.21	5.16	5.19	5.19	5.20
	0.41	0.43	0.46	0.46	0.46	0.22	0.29	0.27	0.21	0.23	0.11	0.15	0.17	0.08	0.04	0.05	0.05	0.03	0.03
0.403	5.99	6.03	6.00	5.98	6.01	5.47	5.43	5.41	5.40	5.37	5.38	5.37	5.36	5.35	5.36	5.35	5.33	5.32	5.32
	0.45	0.47	0.50	0.51	0.51	0.25	0.31	0.30	0.22	0.26	0.13	0.16	0.19	0.09	0.05	0.05	0.05	0.03	0.04
0.516	5.92	5.91	5.96	6.04	6.02	5.48	5.47	5.46	5.44	5.42	5.40	5.39	5.38	5.38	5.36	5.35	5.32	5.30	5.29
	0.45	0.47	0.50	0.51	0.51	0.25	0.31	0.30	0.22	0.26	0.13	0.16	0.19	0.09	0.05	0.05	0.05	0.03	0.04
0.580	6.03	6.09	6.07	6.03	6.08	5.52	5.53	5.53	5.50	5.48	5.48	5.46	5.43	5.44	5.41	5.39	5.37	5.34	5.32
	0.45	0.47	0.50	0.51	0.51	0.25	0.31	0.30	0.22	0.26	0.13	0.16	0.19	0.09	0.05	0.05	0.05	0.03	0.04
0.705	6.21	6.18	6.14	6.18	6.24	5.54	5.54	5.53	5.54	5.50	5.53	5.47	5.47	5.45	5.44	5.44	5.42	5.40	5.38
	0.45	0.47	0.50	0.51	0.51	0.25	0.31	0.30	0.22	0.26	0.13	0.16	0.19	0.09	0.05	0.05	0.05	0.03	0.04
0.787	6.02	6.09	6.09	5.99	6.01	5.32	5.34	5.31	5.28	5.30	5.29	5.29	5.28	5.28	5.27	5.26	5.26	5.24	5.24
	0.41	0.43	0.46	0.46	0.46	0.22	0.29	0.27	0.21	0.23	0.11	0.15	0.17	0.08	0.04	0.05	0.05	0.03	0.03
0.896	6.20	6.19	6.09	6.22	6.36	5.20	5.14	5.12	5.13	5.10	5.17	5.13	5.10	5.12	5.12	5.16	5.13	5.13	5.13
	0.37	0.39	0.42	0.42	0.42	0.20	0.26	0.24	0.19	0.21	0.10	0.13	0.16	0.07	0.04	0.05	0.05	0.03	0.03
0.911	5.97	5.90	5.86	5.77	5.79	5.18	5.14	5.12	5.11	5.12	5.13	5.14	5.11	5.12	5.15	5.15	5.13	5.13	5.13
	0.37	0.39	0.42	0.42	0.42	0.20	0.26	0.24	0.19	0.21	0.10	0.13	0.16	0.07	0.04	0.05	0.05	0.03	0.03
0.977	5.87	5.89	5.88	5.96	5.92	5.04	5.02	5.02	5.02	5.04	5.04	5.02	5.04	5.05	5.05	5.07	5.05	5.06	5.05
	0.37	0.39	0.42	0.42	0.42	0.20	0.26	0.24	0.19	0.21	0.10	0.13	0.16	0.07	0.04	0.05	0.05	0.03	0.03
*	1.19	1.15	1.14	1.12	1.11	1.02	0.99	0.95	0.94	0.93	0.91	0.82	0.77	0.72	0.66	0.64	0.58	0.55	0.50

* Reddening corrections.

The observations for different nights were grouped together according to the phases of the star. The phases have been calculated with the help of the ephemeris (Kukarkin *et al.*, 1969)

$$\text{Phase} = \frac{\text{JD} - 2\,437\,645.789}{1.949\,298}. \tag{1}$$

The observations have been corrected for interstellar reddening. For this purpose a colour excess $E(B-V) = 0\overset{m}{.}24$ has been adopted for SU Cas following Dean *et al.* (1978). The reasons for preferring the reddening values given by Dean *et al.* over other values are discussed by Cogan (1978). Canavaggia *et al.* (1975) and Pel (1978) have also discussed the reddening corrections $E(B-V)$ for cepheids varying from one phase to another, because of different energy distribution. In the case of SU Cas, however, we have used a single value of the colour excess in all the phases, since the changes in the energy distribution curves with phase are small.

By Whitford's (1958) reddening curve, and $A_v = 3E(B-V)$, the reddening corrections for each observed wavelength have been determined. The interstellar reddening law given by Schild (1977) does not seem to affect any appreciable change in the slope of the dereddened curves.

The mean of the absolute visual magnitudes of SU Cas given by Racine (1968), Fernie (1967) and Sandage and Tammann (1968) is $= -2\overset{m}{.}44$. This value is close to luminosity class II of the MK luminosity classes. We have, therefore, adopted the blanketing corrections corresponding to luminosity class II from Ardeberg and Virdefers (1975) to correct the reddening free monochromatic magnitudes of SU Cas for line blanketing. Table I gives the reddening and blanketing free magnitudes of SU Cas along with the values of these corrections. The standard deviation of an individual measurement does not exceed $0\overset{m}{.}03$ shortward of 370.4 nm and $0\overset{m}{.}02$ above 403.6 nm.

3. Effective Temperature

The corrected absolute energy distribution curves obtained as described in the last section were then compared with the model atmospheres computed by Carbon and Gingerich (1969). The slopes between $1/\lambda = 1.41$ and $2.35\,\mu\text{m}^{-1}$ of the model atmosphere curves for $\log g = 2$ were plotted against the effective temperature. Corresponding slopes of the energy distribution curves at different phases of SU Cas were obtained through a least-squares fit of the observed points, and the effective temperatures were read off from the slope-T_e curve for the models. At phase 0.072 the observations are not available up to $1/\lambda = 1.41\,\mu\text{m}^{-1}$. At this phase, therefore, the slope has been taken between $1/\lambda = 1.71$ and $2.35\,\mu\text{m}^{-1}$ only. The error in the temperature estimates on account of the deviations in the slopes determined by least-square fitting does not exceed ± 60 K. We have adopted a value of $\log g = 2$ for the star consistent with its luminosity class. Also the $\log g$ values given for the star by

TABLE II

Effective temperature, displacement and radius of SU Cas according to the phase of light variation

Phase	T_e K	θ_e	$(r - \bar{R})$ 10^{10} cm	$R/R_\odot$
0.072	6820	0.739	−0.07	21.75
0.189	6790	0.742	2.52	22.12
0.254	6750	0.747	3.56	22.27
0.403	6580	0.766	4.60	22.42
0.516	6410	0.786	3.82	22.31
0.580	6330	0.796	2.84	22.17
0.705	6410	0.786	0.39	21.82
0.787	6620	0.761	−1.27	21.58
0.896	6770	0.744	−2.56	21.39
0.911	6800	0.741	−2.57	21.39
0.977	6860	0.735	−2.12	21.46

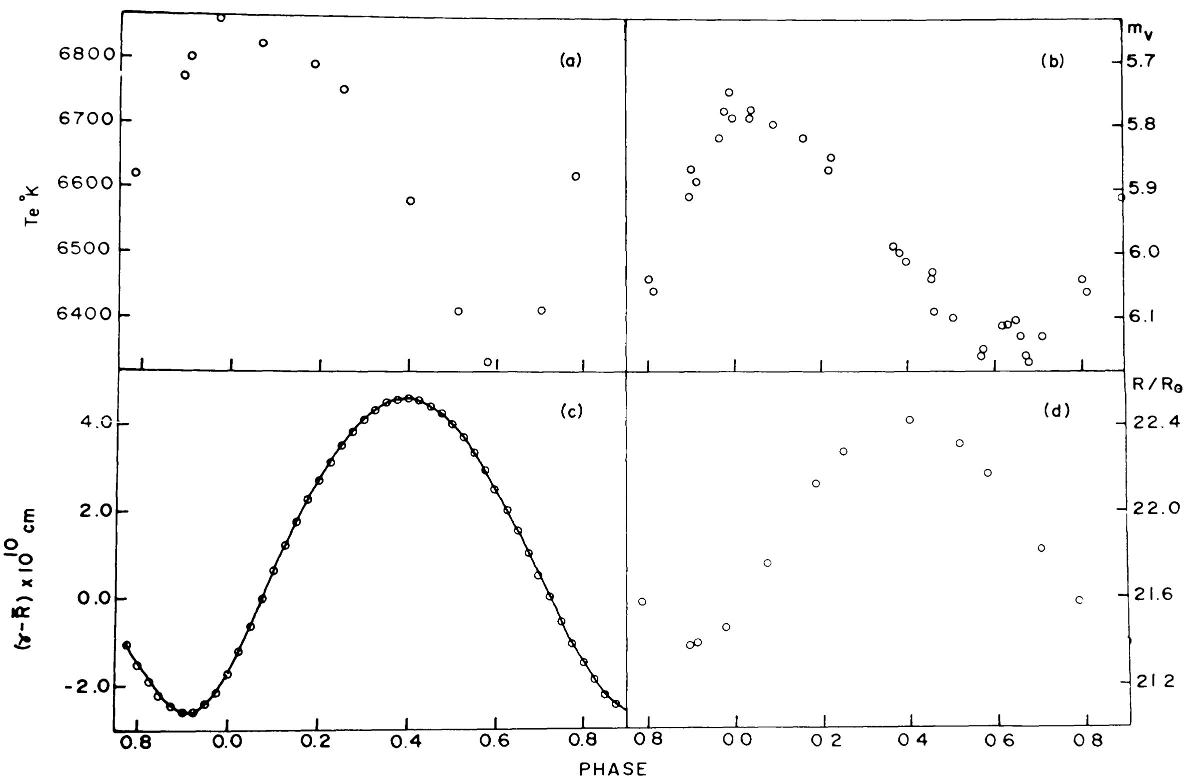

Fig. 1. Variation with phase in the parameters of SU Cas (a) effective temperature, (b) visual magnitude, (c) displacement, and (d) radius.

Parsons (1970) based upon six colour photometry, range between $\log g = 2$ to $\log g = 2.5$. The error in the temperature estimates on this account can reach a maximum of 100 K, in the range of temperatures found for the star. The temperatures at different phases of the star thus obtained are given in Table II and are plotted in Figure 1(a). A mean effective temperature of 6650 K is obtained for SU Cas. This value of T_e can be compared with the value of 6599 K given by Gieren (1976) derived from the $T_e - (B - V)_0$ relation. The six colour photometry by Parsons (1970) gives a mean effective temperature of 6390 K, whereas the H_α profile analysis by Schmidt (1972) resulted in an effective temperature of 6667 K for the star.

4. Radius

Though several values of radius determinations of SU Cas are available (Gieren, 1976; Balona, 1977; and references in Cogan, 1978), the need for a refinement in the radius and possibly the luminosity of the star continues to be urgently felt (Cox, 1979). We have in this case used the radial velocity curve given by Abt (1959) and the V light curve (Figure 1(b)) given by Mitchell *et al.* (1964) to derive a Wesselink-radius for the star. The integration of the radial velocity curve, converted to photospheric pulsation velocity curve by using the factor 24/17, gives the displacement $r - \bar{R}$ (Figure 1(c)) at several phases of the star, where $\bar{R}$ is the mean radius and r is the instantaneous value of the radius at the corresponding phase. Phases of equal temperature have been determined from Figure 1(a). Seven pairs of equal temperature phases were marked and the corresponding differences in magnitude were read off from the V light curve given by Mitchell *et al.* (1964). The mean radius of the star was determined with the help of each of the seven phase pairs using the relation

$$\left(\frac{\bar{R} + \Delta r_1}{\bar{R} + \Delta r_2}\right)^2 = 10^{0.4(\Delta m)}, \tag{2}$$

where Δr_1 and Δr_2 are obtained from the displacement curve and Δm is the V magnitude difference at the two phases of equal effective temperature. The average value for the mean radius of the star comes out at 21.76 $R_\odot \simeq 22$ $R_\odot$. The radius variation in a light cycle is shown in Figure 1(d). The displacement and radius values at 11 phases of the star are given in Table II.

5. Luminosity

Using the radius and effective temperature determined above, the luminosity of SU Cas has been determined. We obtain M_{bol} equal to $-2\overset{\text{m}}{.}98$ for the star, corresponding to $\log L/L_\odot = 2.93$.

6. Mass

The Wesselink-radius mass of the star has been determined using the radius determined in this paper and the relations

$$Q_i = \pi(\langle \varrho \rangle / \langle \varrho_\odot \rangle)^{1/2} \tag{3}$$

and

$$\log Q_i - g_i(m) = a_i \exp\{-b_i[\log(M/R) - c_i]^2\} + d_i, \tag{4}$$

where

$$g_0(m) \equiv A_0 m. \tag{5}$$

The values of A_0, a_i, b_i, c_i, d_i have been taken from the table of Cox *et al.* (1972) for fundamental mode pulsation of the star. By use of assumed values of M/R, Q_0 was calculated and a period of pulsation determined. The process was repeated until a period equal to the period of SU Cas was obtained. A value of 3.65 $M_\odot$ is obtained for the Wesselink-radius mass of the star.

An evolutionary mass for the star has been estimated by using Becker *et al.* (1977) evolutionary tracks and their mass luminosity relation given by

$$\log L = j + k \log M, \tag{6}$$

where

$$j = 0.46 - 41\,\Delta z + 6.6\,\Delta y - 5(\Delta y)^2 \tag{7}$$

and

$$k = 3.68 + 21\,\Delta z - 4.5\,\Delta y + 11(\Delta y)^2, \tag{8}$$

in which Δz and Δy are assumed to be zero corresponding to $y = 0.28$ and $z = 0.02$. Using the luminosity for SU Cas as determined above in Equation (6), we get a value of $M_{\text{evo}} = 4.64\ M_\odot$ for the star. The ratio $M_{\text{evo}}/M_{\text{wess}} = 1.27$. Values of $y = 0.31$, $z = 0.03$ lead to a value of $M = 5.1\ M_\odot$. Also by using the relation

$$P_0 = 0.025(M/M_\odot)^{-0.67}(R/R_\odot)^{1.70} \tag{9}$$

given by Fricke *et al.* (1972) for the fundamental period P_0 (in days) for standard population I composition, we have calculated the mass for SU Cas using $P_0 = 1.95$ days as the fundamental mode period. A value of $M = 3.72\ M_\odot$ is thus obtained. Gieren (1976) obtains a value $M = 7.71\ M_\odot$ with his value of $R = 29.0\ R_\odot$ and thereby obtains a ratio $M_{\text{evo}}/M_{\text{pul}} = 0.56$, which according to him can be reconciled with the well-known mass discrepancy for cepheids by assuming that the observed period of 1.95 days is the first harmonic mode period of the star.

References

Abt, H. A.: 1959, *Astrophys. J.* **130**, 1021.

Ardeberg, A. and Virdefers, B.: 1975, *Astron. Astrophys.* **39**, 21.

Balona, L. A.: 1977, *Monthly Notices Roy. Astron. Soc.* **178**, 231.

Becker, S. A., Iben, I., Jr. and Tuggle, R. S.: 1977, *Astrophys. J.* **218**, 633.

Canavaggia, R., Mianes, P. and Rousseau, J.: 1975, *Astron. Astrophys.* **43**, 275.

Carbon, D. F. and Gingerich, O.: 1969, in O. Gingerich (ed.), *Proc. Third Harvard-Smithsonian Conference on Stellar Atmospheres*, p. 377, Cambridge.

Cogan, B. C.: 1978, *Astrophys. J.* **221**, 635.

Cox, A. N.: 1979, *Astrophys. J.* **229**, 212.

Cox, J. P., King, D. S. and Stellingwerf, R. F.: 1972, *Astrophys. J.* **171**, 93.

Dean, J. F., Warren, P. R. and Cousins, A. W.: 1978, *Monthly Notices Roy. Astron. Soc.* **183**, 569.

Fernie, J. D.: 1967, *Astron. J.* **72**, 422.

Frike, K., Stobie, R. S. and Strittmatter, P. A.: 1972, *Astrophys. J.* **171**, 593.

Gieren, W.: 1976, *Astron. Astrophys.* **47**, 211.

Hayes, D. S. and Latham, D. W.: 1975, *Astrophys. J.* **197**, 593.

Kukarkin, B. V., Kholopov, P. N., Efremov, Yu. N., Kukarkin, N. P., Kurochkin, N. E., Medvedeva, G. I., Perova, N. B., Fedorovich, V. P. and Frolov, M. S.: 1969, *General Catalogue of Variable Stars*, 3rd edition, Moscow.

Mitchell, R. I., Iriarte, B., Steinmetz, D. and Johnson, H. L.: 1964, *Bull. Obs. Tonantzintla Tacubaya*, **3**, 153.

Oke, J. B.: 1965, *Ann. Rev. Astron. Astrophys.* **3**, 23.

Parsons, S. B.: 1970, *Warner and Swasey Observatory*, *Preprint No. 3.*

Pel, J. W.: 1978, *Astron. Astrophys.* **62**, 75.

Racine, R.: 1968, *Astron. J.* **73**, 588.

Rautela, B. S. and Joshi, S. C.: 1976, *Astrophys. Space Sci.* **40**, 455.

Sandage, A. and Tammann, G. A.: 1968, *Astrophys. J.* **151**, 531.

Schild, R. E.: 1977, *Astron. J.* **82**, 337.

Schmidt, E. G.: 1972, *Astrophys. J.* **174**, 595.

Whitford, A. E.: 1958, *Astron. J.* **63**, 201.