Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **403,** 1491–1506 (2010) doi:10.1111/j.1365-2966.2009.16213.x

Integrated parameters of star clusters: a comparison of theory and observations

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Accepted 2009 December 14. Received 2009 November 18; in original form 2009 March 17

ABSTRACT

This paper presents integrated magnitudes and colours for synthetic clusters. The integrated parameters have been obtained for the whole cluster population as well as for the main-sequence (MS) population of star clusters. We have also estimated observed integrated magnitudes and colours of the MS population of galactic open clusters, Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC) star clusters. It is found that the colour evolution of the MS population of star clusters is not affected by the stochastic fluctuations, however, these fluctuations significantly affect the colour evolution of the whole cluster population. The fluctuations are maximum in $(V - I)$ colour in the age range $6.7 < \log(\text{age}) < 7.5$. Evolution of integrated colours of the MS population of clusters in the Milky Way, LMC and SMC, obtained in the present study is well explained by the present synthetic cluster model. The observed integrated $(B - V)$ colours of the MS population of LMC star clusters having age ≥500 Myr seem to be distributed around the $Z = 0.004$ model, whereas $(V - I)$ colours are found to be bluer than those predicted by the $Z = 0.004$ model. The $(V - I)$ versus $(B - V)$ two-colour diagram for the MS population of the Milky Way star clusters shows a fair agreement between the observations and present model, however, the diagrams for LMC and SMC clusters indicate that observed $(V - I)$ colours are relatively bluer. Possible reasons for this anomaly have been discussed. Comparison of the synthetic $(U - B)$ versus $(B - V)$ relation with the observed integrated parameters of the whole cluster population of the Milky Way, LMC and SMC star clusters indicates that the majority of the bluest clusters $[(B - V)_0 < 0.0]$ follow the MS population relation. The colour evolution of young Milky Way, LMC and SMC clusters $[6.5 \leq \log(\text{age}) \leq 8.0]$ also indicates that a large number of young clusters follow the MS population relation. Therefore, in the absence of a careful modelling of stochastic effects, age determination of young star clusters by comparing their integrated colours with whole cluster synthetic colours may yield erroneous results.

Key words: open clusters and associations: general – galaxies: star clusters.

1 INTRODUCTION

Star clusters are useful objects to test the theories of stellar evolution and stellar dynamics. The star clusters in the Milky Way and in the Magellanic Clouds (MCs) span a wide range in age (from a few Myr to a few Gyr). The large range in the age of the clusters allows us to observe star clusters at various epochs in their evolution and makes it possible to identify evolutionary trends.

In spite of the advent of new generation ground and space based telescopes, the integrated parameters of star clusters are the only observable parameters to investigate the evolutionary history of stellar systems beyond the local group of galaxies. In order to interpret the integrated parameters of extragalactic star clusters, it is necessary to study the integrated parameters of star clusters of our Galaxy where observation of individual stars in the cluster region can be carried out to study the various parameters like age, mass, metallicity, etc. with sufficient accuracy (e.g. Hancock et al. 2008, and references therein).

Various efforts both from a theoretical and from an observational point of view (see e.g. Searle, Wilkinson & Bagnuolo 1980; Sagar, Joshi & Sinvhal 1983; Chiosi, Bertelli & Bressan 1988; Pandey et al. 1989; Battinelli, Brandimarti & Capuzzo-Dolcetta 1994; Brocato et al. 1999; Lata et al. 2002; Bruzual & Charlot 2003) have

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been made to interpret the integrated photometric colours of a simple stellar population (SSP), like star clusters, in terms of stellar ages and chemical composition. Lata et al. (2002) have calculated integrated magnitudes and colours of 140 open clusters of the Milky Way which in combination with earlier estimates provide integrated photometric parameters for 352 star clusters. Lata et al. (2002) have reported for the first time integrated $(V-I)$ and $(V-R)$ colours for 58 and 23 star clusters, respectively. Although the integrated U, B, V magnitudes reported by them are for the whole population [main sequence (MS)+ evolved cluster population], the integrated R and I magnitudes for clusters younger than 100 Myr are mainly for the MS population as CCD observations of bright stars are not available in these clusters. Therefore, the observed evolution of $(V-R)$ and $(V-I)$ colours is different from the theoretical evolution (which is for the whole cluster population) given by Maraston (1998) and Brocato et al. (1999).

Integrated *UBV* photometry for 147 Large Magellanic Cloud (LMC) star clusters was reported by van den Bergh (1981). The sample of integrated *UBV* magnitudes was further enhanced to 624 by Bica et al. (1992, 1996). Recently, Rafelski & Zaritsky (2005, hereafter RZ05) have reported integrated magnitude and colours for 195 Small Magellanic Cloud (SMC) star clusters. A comparison of their data with the model of Leitherer et al. (1999) and Anders & Fritze-v. Alvensleben (2003) indicates a large scatter in the observed data with a systematic difference between the observed data and the model.

Because of the relevance of the integrated parameters, population synthesis models have been continuously upgraded over the years (cf. Brocato et al. 1999; Anders & Fritze-v. Alvensleben 2003, and references therein). Similarly, the catalogue of the observed integrated parameters of galactic open clusters and MC clusters is frequently being upgraded. In the present study, we have made an attempt to append the data base of integrated parameters of the MS population of open clusters, LMC and SMC clusters. Here, we present integrated magnitude and colours of the MS population of 66 galactic open clusters, 745 LMC clusters and 238 SMC clusters. The $(B-V)$, $(V-R)$ and $(V-I)$ colours for the MS population of star clusters in the LMC and SMC are being reported for the first time. Presently, available integrated $(V-R)$ and $(V-I)$ colours of MC star clusters are for the whole population of the cluster. The integrated colours are frequently used to date the clusters (e.g. Elson & Fall 1985; Chiosi et al. 1988; RZ05; Hancock et al. 2008). Since the integrated colours of the whole cluster population are severely affected by the stochastic fluctuation (cf. Chiosi et al. 1988; Section 2 of the present study), the age calibration of the clusters will also be affected accordingly. As the colour evolution of the MS population is quite systematic, therefore the integrated colours of the MS population should give a better estimate of the age of the clusters. In the present study, we have also calculated integrated parameters for the MS and whole population of the synthetic clusters. The comparison with the observational data of galactic open clusters and MC star clusters has also been carried out. The paper is organized as follows. In Section 2, a detailed description of the model is presented. Section 3 describes the estimation of observed integrated parameters of star clusters. In Sections 4 and 5, a comparison of observed and theoretical integrated parameters has been carried out. Section 6 describes the conclusion of the present study.

Table 1. Integrated magnitude and colours of synthetic clusters (MS population) obtained in the present work.

MF slope & metallicity	Age (log t)	$(U-V)_0$ (mag)	err (mag)	$(U-B)_0$ (mag)	err (mag)	$(B-V)_0$ (mag)	err (mag)	$(V-R)_0$ (mag)	err (mag)	$(V-I)_0$ (mag)	err (mag)
$X=2.35$	7.8	−0.776	0.007	0.647	0.005	−0.129	0.002	−0.049	0.001	−0.113	0.002
$Z=0.001$	7.9	−0.714	0.005	−0.599	0.004	−0.115	0.002	−0.042	0.001	−0.097	0.002
	8.0	−0.645	0.005	−0.546	0.003	−0.099	0.002	−0.034	0.001	−0.078	0.002
	8.1	−0.567	0.005	−0.488	0.003	−0.080	0.002	−0.024	0.001	−0.055	0.002
	8.2	−0.520	0.005	−0.449	0.003	−0.071	0.002	−0.020	0.001	−0.044	0.002
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$X=2.35$	6.6	−1.283	0.020	−1.040	0.015	−0.243	0.005	−0.102	0.002	−0.240	0.006
$Z=0.004$	6.7	−1.259	0.020	−1.023	0.015	−0.237	0.005	−0.100	0.002	−0.234	0.005
	6.8	−1.248	0.015	−1.013	0.011	−0.235	0.004	−0.099	0.002	−0.233	0.004
	6.9	−1.144	0.018	−0.934	0.013	−0.210	0.004	−0.088	0.002	−0.205	0.005
	7.0	−1.126	0.015	−0.920	0.011	−0.206	0.004	−0.086	0.002	−0.201	0.004
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...	...	...	...	...	...	...	...	...	...	...	...
$X=2.35$	6.6	−1.260	0.019	−1.025	0.014	−0.235	0.005	−0.099	0.002	−0.234	0.005
$Z=0.008$	6.7	−1.243	0.019	−1.011	0.014	−0.232	0.005	−0.098	0.002	−0.231	0.005
	6.8	−1.227	0.016	−0.998	0.012	−0.229	0.004	−0.097	0.002	−0.228	0.005
	6.9	−1.112	0.017	−0.911	0.013	−0.201	0.004	−0.083	0.002	−0.196	0.005
	7.0	−1.097	0.010	−0.899	0.008	−0.198	0.002	−0.082	0.001	−0.192	0.003
...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...
$X=2.35$	6.6	−1.226	0.020	−1.001	0.015	−0.225	0.005	−0.095	0.002	−0.226	0.006
$Z=0.02$	6.7	−1.230	0.019	−1.004	0.015	−0.227	0.005	−0.096	0.002	−0.227	0.006
	6.8	−1.201	0.011	−0.982	0.009	−0.219	0.003	−0.092	0.001	−0.219	0.003
	6.9	−1.077	0.009	−0.886	0.007	−0.191	0.002	−0.078	0.001	−0.184	0.003
	7.0	−1.048	0.015	−0.863	0.012	−0.184	0.004	−0.074	0.002	−0.176	0.004
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Note. The complete table is available in electronic form only (see Supporting Information).

Table 2. Integrated magnitude and colours of synthetic clusters (whole population) obtained in the present work.

MF slope & metallicity	Age (log *t*)	$(U-V)_0$ (mag)	err (mag)	$(U-B)_0$ (mag)	err (mag)	$(B-V)_0$ (mag)	err (mag)	$(V-R)_0$ (mag)	err (mag)	$(V-I)_0$ (mag)	err (mag)
$X=2.35$	7.8	−0.353	0.106	−0.399	0.063	0.045	0.075	0.097	0.063	0.236	0.142
$Z=0.001$	7.9	−0.288	0.115	−0.353	0.062	0.065	0.073	0.107	0.059	0.258	0.130
	8.0	−0.232	0.105	−0.310	0.047	0.078	0.072	0.113	0.059	0.269	0.130
	8.1	0.031	0.249	−0.214	0.064	0.244	0.191	0.251	0.163	0.559	0.340
	8.2	0.138	0.280	−0.164	0.083	0.302	0.204	0.293	0.171	0.647	0.354
	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...
$X=2.35$	6.6	−1.352	0.008	−1.091	0.006	−0.261	0.002	−0.111	0.001	−0.261	0.003
$Z=0.004$	6.7	−1.332	0.011	−1.078	0.007	−0.254	0.004	−0.108	0.002	−0.256	0.004
	6.8	−0.859	0.289	−0.722	0.212	−0.137	0.079	−0.038	0.048	−0.090	0.117
	6.9	−0.873	0.353	−0.769	0.214	−0.104	0.167	−0.009	0.124	−0.026	0.279
	7.0	−0.601	0.331	−0.677	0.168	0.076	0.223	0.146	0.177	0.329	0.391
	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...
$X=2.35$	6.6	−1.346	0.009	−1.089	0.006	−0.257	0.002	−0.109	0.001	−0.260	0.003
$Z=0.008$	6.7	−1.117	0.256	−0.949	0.117	−0.168	0.153	−0.052	0.107	−0.133	0.235
	6.8	−0.524	0.513	−0.598	0.290	0.074	0.250	0.093	0.155	0.189	0.332
	6.9	−0.827	0.413	−0.817	0.187	−0.010	0.261	0.063	0.191	0.125	0.418
	7.0	−0.797	0.268	−0.765	0.139	−0.032	0.167	0.077	0.158	0.185	0.372
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	...	...	...	...	...	...	...	...	...	...	...
$X=2.35$	6.6	−1.317	0.008	−1.072	0.005	−0.245	0.003	−0.105	0.002	−0.252	0.004
$Z=0.02$	6.7	−1.083	0.237	−0.925	0.117	−0.158	0.130	−0.049	0.086	−0.127	0.186
	6.8	−0.779	0.331	−0.750	0.180	−0.029	0.195	0.049	0.153	0.093	0.334
	6.9	−0.693	0.426	−0.774	0.194	0.081	0.240	0.137	0.173	0.299	0.391
	7.0	−0.640	0.321	−0.789	0.115	0.149	0.211	0.254	0.169	0.601	0.386
	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...

Note. The complete table is available in electronic form only (see Supporting Information).

2 THEORETICAL INTEGRATED PARAMETERS OF STAR CLUSTERS

Various evolutionary population synthesis (EPS) models, as mentioned in Section 1, have been developed for SSPs. The results of EPS may differ from one another due to input parameters. The comparison of EPS with observations can give information about acceptability of a particular EPS (see e.g. Lata et al. 2002).

We have generated synthetic colour–magnitude diagram (CMDs) of open clusters (Sandhu, Pandey & Sagar 2003) using stellar evolutionary models by Girardi et al. (2002). The synthetic CMDs are constructed using the technique described by Chiosi et al. (1989). Briefly, this technique consists of random generation of stars by means of a Monte Carlo technique and distributing the stars along a given isochrone according to evolutionary phases and the initial mass function (IMF). The following expression is used to describe the IMF:

$$\mathrm{d}N = AM^{-X}\mathrm{d}M,$$

where dN is the number of stars in the mass interval dM, X is the slope of the mass function. The Salpeter (1955) value for the slope of the mass function is 2.35. The constant A is fixed in such a way that the initial mass of cluster stars having masses between $0.6 \le \mathrm{M}_{\odot} \le 40$ is about 4000 $\mathrm{M}_{\odot}$ (for details see Sandhu et al. 2003). The initial mass value of 4000 $\mathrm{M}_{\odot}$ is selected as it represents the approximate average mass of LMC clusters (see e.g. Girardi & Bica 1993, their fig. 8). The contribution of binary content has not been taken into account. The star formation is assumed to be instantaneous. The integrated magnitudes and colours for the whole cluster population (i.e. MS and red giant population) as well as for the MS population were calculated using the procedure described by Pandey et al. (1989). One hundred simulations at log(age) interval of 0.1 were carried out (for various combinations of the metallicity $Z = 0.001$, 0.004, 0.008 and 0.02 and mass function slope $X = 1.0$, 1.35, 2.35 and 3.35) and then averaged. In the case of $Z = 0.001$, we have also used the model by Bertelli et al. (1994) as the model by Girardi et al. is available for log age ≥ 7.6. The integrated magnitudes and colours of synthetic clusters having Salpeter mass function and various assumed values of metallicity are given in Tables 1 and 2. A sample of the tables is shown here. Complete tables are available in electronic form only.

2.1 The evolution of integrated magnitudes and colours

In Fig. 1, we show the evolution of U, B, V, R and I magnitudes of a synthetic cluster having solar metallicity $Z = 0.02$ and classical Salpeter mass function $X = 2.35$. The error bars show the 1σ dispersion of the average results obtained from 100 independent simulations. The I band integrated magnitudes for the synthetic cluster having evolved stars show the largest errors. In both the cases (i.e. whole population and MS population), integrated luminosity drops in all bands because of the disappearance of bright MS and supergiant stars. The stochastic fluctuations are relatively less important in the case of MS populations.

The $(U-B)$, $(U-V)$, $(B-V)$, $(V-R)$ and $(V-I)$ colour evolution along with standard deviation for the cluster having $X = 2.35$ and $Z = 0.02$ is shown in Fig. 2. All five colours in

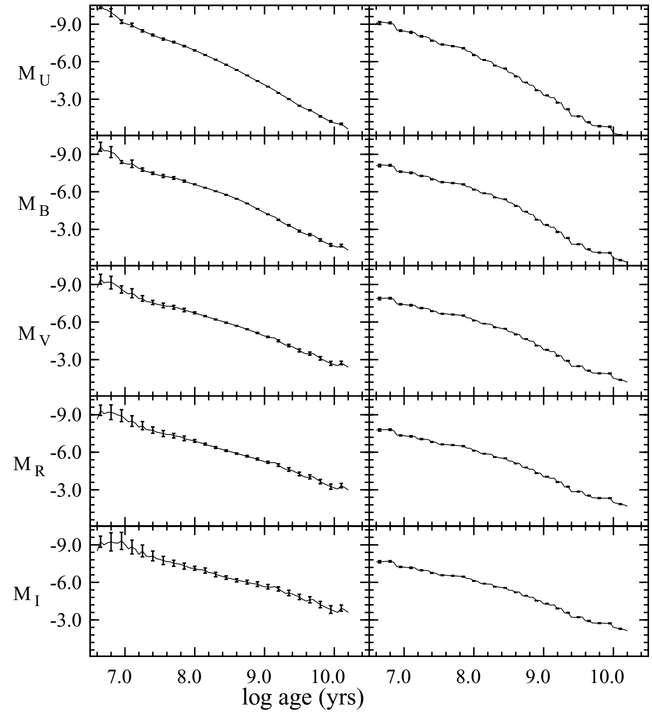

Figure 1. Time evolution of integrated U, B, V, R and I magnitudes of a cluster having $X = 2.35$ and $Z = 0.02$. The left-hand panel shows evolution of integrated magnitude of the whole cluster population, whereas the right-hand panel shows results for MS stars only.

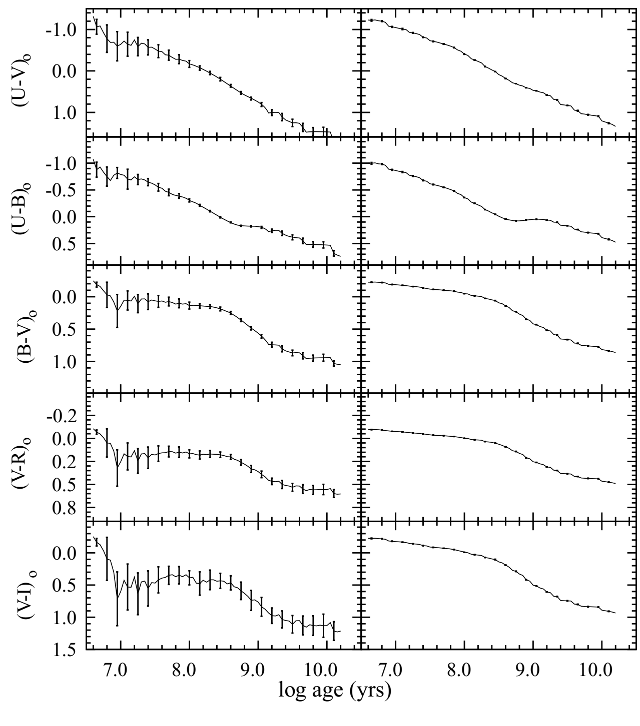

Figure 2. The left- and right-hand panels show time evolution of integrated colours of the whole cluster population and MS population, respectively, having $X = 2.35$ and $Z = 0.02$.

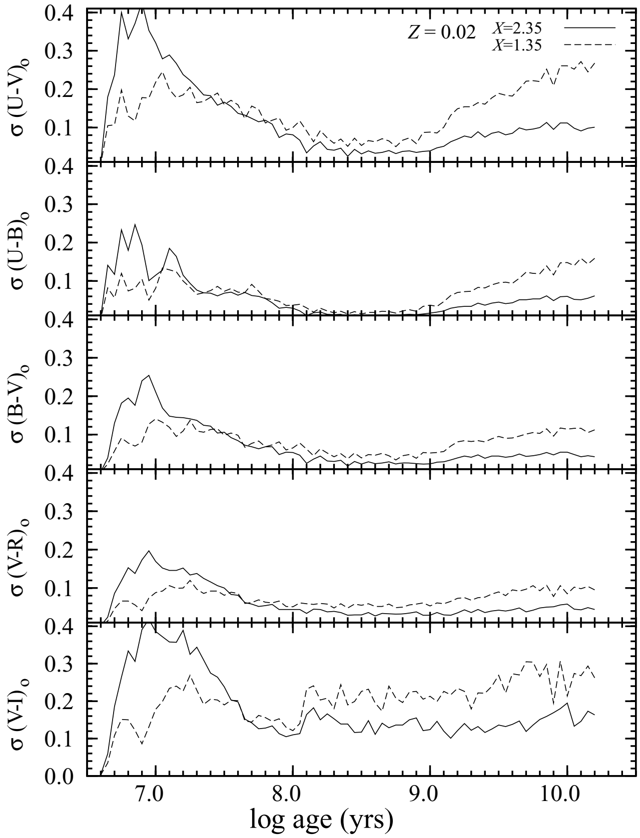

Figure 3. Effect of stochastic fluctuations on colours of the synthetic cluster having a post-MS population.

the case of the MS population vary smoothly with age, whereas in the case of clusters having an evolved population, only $(U - B)$ and $(U - V)$ colour vary smoothly with age. The $(B - V)$, $(V - I)$, $(V - R)$ colours show a plateau around 10 to 200 Myr. In the case of an evolved population, especially in the $(V - I)$ colour, the stochastic fluctuations are relatively larger than those obtained for the MS population only. The colour variation with the age of the cluster, during 10 to 1000 Myr, is maximum in the $(U - B)$, $(U - V)$ colours, whereas the variation is minimum for $(V - R)$ colour.

2.2 Effect of stochastic fluctuations on colours

Stochastic effects can produce a significant amount of dispersion in the integrated colours, especially in the integrated colours of very young clusters which contain RSG stars (cf. Girardi et al. 1995). To study the influence of stochastic effects on colour evolution of synthetic clusters, we carried out 100 simulations assuming $Z = 0.02$ and $X = 2.35$ and 1.35 for the cluster. We estimated the mean colour and standard deviation around the mean colours.

Fig. 3 displays the influence of stochastic fluctuations on integrated colour as a function of age which indicates the following.

(i) For $\log(\text{age}) \leq 6.7$, the dispersion is low in all the colours. This is due to lack of evolved stars.

(ii) For $6.7 \leq \log(\text{age}) \leq 7.5$, the dispersion is high with a peak at $\log(\text{age}) \sim 7.0$. The large scatter in the colours during this period is due to the small number of RSG stars. The dispersion in $(V - I)$ is relatively higher at all ages.

(iii) In the case of clusters having $\log(\text{age}) > 7.5$, the dispersion in colours decreases with the age.

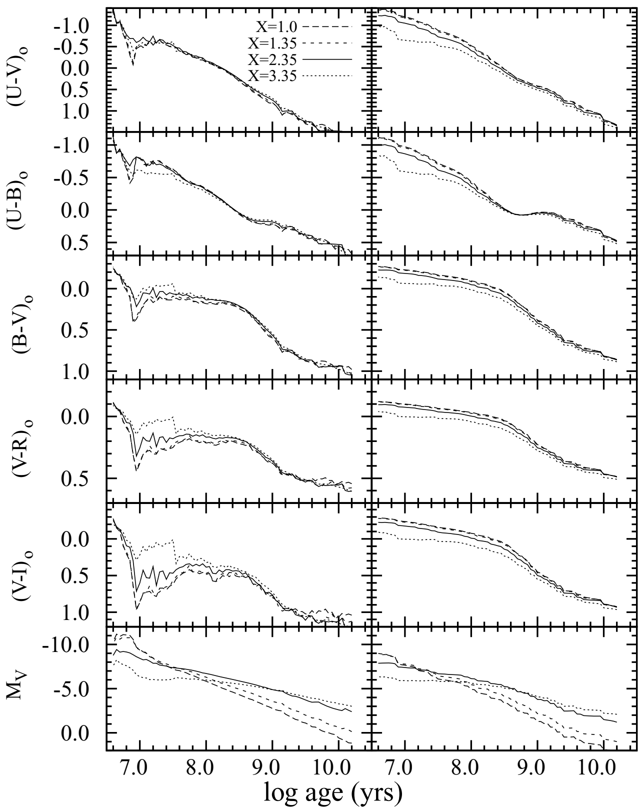

Figure 4. Left-hand panel: evolution of integrated V magnitude and colours of synthetic clusters (whole population) having $Z = 0.02$ and $X = 1.0$, 1.35, 2.35, 3.35. Right-hand panel: same as left-hand panel, but for the MS population.

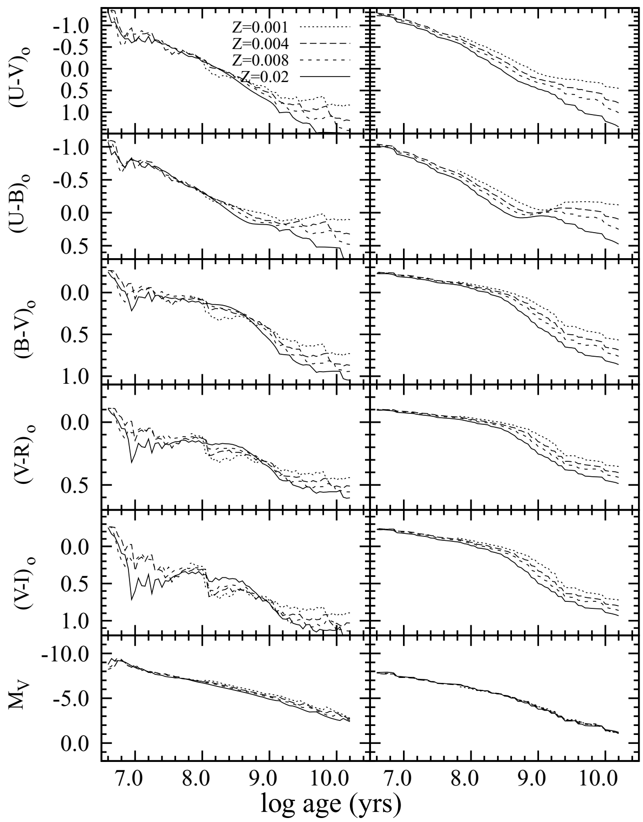

Figure 5. Left-hand panel: effect of metallicity on the integrated magnitude and colours of synthetic clusters (whole population) having a Salpeter mass function and $Z = 0.004$, 0.008, 0.02. Right-hand panel: same as left-hand panel, but for the MS population.

2.3 Effect of IMF and chemical composition

To study the influence of the IMF on the integrated magnitude and colours, we again carried out simulations by varying the slope of the mass function X. Fig. 4 shows the evolution of synthetic V magnitude and colours for $X = 1.0$, 1.35, 2.35, 3.35. For smaller values of X, a large fraction of stars comes from massive stars. Hence, integrated magnitudes of the cluster become brighter. However, for ages greater than 1000 Myr, the larger value of X results in fainter clusters because at ages greater than 1000 Myr, the luminosity contribution comes from less massive (i.e. fainter) stars. The evolution of integrated colours of clusters having log(age) ≥ 7.5 is not affected by the variation of the IMF. However, the colour evolution for log(age) < 7.5 is influenced by the IMF. The dispersion is maximum in the $(V - I)$ colour of the whole cluster population. Whereas the effect of the IMF is less prominent on the integrated $(V - I)$ colours of the MS population of the clusters.

The influence of metallicity on the integrated magnitude and colours has been studied assuming $Z = 0.004$, 0.008, 0.02 and shown in Fig. 5. The metallicity variation does not show any significant impact on the evolution of integrated V magnitude as well as on the $(U - B)$, $(U - V)$, $(B - V)$, $(V - R)$ and $(V - I)$ colour evolution of clusters having log(age) <8.5, however, for older clusters the colours become bluer for $Z = 0.008$ and 0.004 than the solar metallicity models. The same effect has also been reported and discussed by Brocato et al. (1999). However, the effect of metallicity on the $(B - V)$ colour obtained by us is not so prominent as reported by Brocato et al. (1999).

2.4 Comparison with previous models

The EPS models have been continuously upgraded over the years (e.g. Maraston 1998; Brocato et al. 1999). The comparison of different EPS models is not a simple task, because many factors contribute to produce different results (cf. Maraston 1998). Maraston (1998) and Brocato et al. (1999) have compared various EPS models and concluded that the evolution of $(U - B)$ and $(B - V)$ is very similar for all sets of models, whereas in the case of near-infrared indices at intermediate ages a major source of discrepancy between the models arises due to the asymptotic giant branch (AGB) phase.

In Fig. 6, we compare our results for solar metallicity and a Salpeter IMF (whole cluster population) with those by Brocato et al. (1999) and Maraston (1998). The comparison indicates the following.

(i) The $(U - B)$ and $(U - V)$ colours obtained in the present work are slightly bluer in comparison to those given by Brocato et al. (1999) and Maraston (1998).

(ii) An agreement can be seen between $(B - V)$ colours obtained in the present work and those by Brocato et al. (1999). Keeping in mind the errors reported in Section 2.1, the $(B - V)$ colours obtained in the present work are in reasonable agreement with those given by Maraston (1998).

(iii) For synthetic clusters having age >15 Myr an agreement can be seen between $(V - R)$ colour evolution obtained in the present work and that predicted by Brocato et al. (1999). For clusters younger than $\sim$15 Myr, the $(V - R)$ colours obtained in the present work are bluer than the colours predicted by Brocato

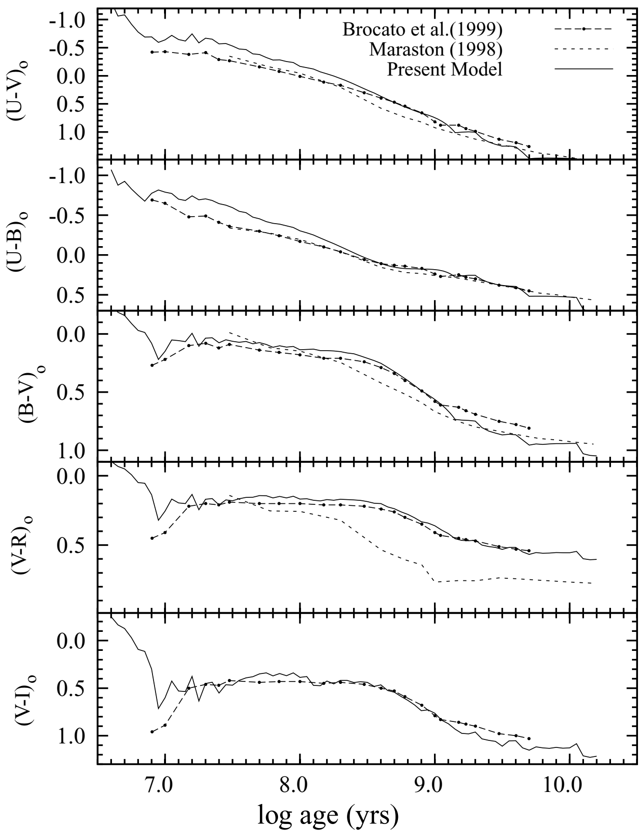

Figure 6. Comparison of integrated colours (for $X = 2.35$, $Z = 0.02$) obtained in the present work with those obtained by Brocato et al. (1999) and Maraston (1998).

et al. (1999). On the other hand, the evolution of $(V - R)$ colour predicated by Maraston (1998) does not match either with the present work or with the work of Brocato et al. (1999). The $(V - R)$ colours by Maraston (1998) are significantly redder than those obtained in the present work or by Brocato et al. (1999).

(iv) Considering the errors in the present $(V - I)$ colour estimation (cf. Fig. 2), the predicted $(V - I)$ colour evolution of clusters in the present work and that by Brocato et al. (1999) are in fair agreement.

3 OBSERVED INTEGRATED PARAMETERS

3.1 Galactic clusters

Using the observation of individual stars of a galactic star cluster, integrated photometric parameters have been obtained by several authors (Lata et al. 2002, and references therein). Lata et al. (2002) have obtained integrated parameters of 140 clusters, which in combination with earlier estimates provide integrated photometric parameters for 352 clusters. Lata et al. (2002) have reported integrated $(V - R)$ and $(V - I)$ colours for the first time, however, for most of the younger clusters (age $\leq$100 Myr), the integrated $(V - R)$ and $(V - I)$ may represent the MS population because young clusters have bright stars for which generally $(V - R)$ and $(V - I)$ CCD observations are not available. In the present work, we have estimated integrated colours for only the MS population of galactic clusters. The *UBV* Johnson and *RI* Cousins CCD data along with the distance, $E(B - V)$ and age of the galactic clusters have been taken from the WEBDA data base (http://www.univie.ac.at/webda). The integrated magnitude and colours were calculated using the procedure described by Pandey et al. (1989). The colour excesses $E(U - B)$, $E(U - V)$, $E(V - R)$ and $E(V - I)$ have been calculated from $E(B - V)$ using the relations $E(U - B) = 0.72E(B - V)$, $E(U - V) = 1.72E(B - V)$, $E(V - R) = 0.60E(B - V)$ and $E(V - I) = 1.25E(B - V)$. The possible source of errors in determination of the integrated parameters is same as described by Sagar et al. (1983). The uncertainty in estimation of integrated magnitude and colours is ∼0.5 and ∼0.2 mag, respectively. Battinelli et al. (1994) have also reported the same order of uncertainty in their estimation of integrated parameters. A comparison of integrated parameters obtained by various authors has been carried out by Lata et al. (2002, their fig. 1), which clearly supports the reported errors of 0.5 mag in estimation of observed integrated magnitudes.

Table 3 gives a sample of the catalogue of intrinsic integrated M_V magnitude and $(U - B)$, $(U - V)$, $(B - V)$, $(V - R)$ and $(V - I)$ colours. The complete catalogue is available in electronic form only.

3.2 Clusters in the Magellanic Cloud

Integrated *UBV* photometry for 147 LMC clusters has been reported by van den Bergh (1981). Bica et al. (1992, 1996) extended the sample of LMC clusters to a total of 624 objects. Their sample includes fainter clusters and they claim that the catalogue is complete up to $V \approx 13.2$. Recently, Hunter et al. (2003) have studied integrated properties of 939 star clusters in the MC, which were based on ground based CCD images in *UBVR* passbands. All of the studies mentioned above are based on integrated photometry of the clusters, therefore the integrated parameters are for the whole population (i.e. MS + giant stars) of the clusters. To our knowledge, none of the studies is available in the literature where integrated magnitude and colours for only the MS population of MC star clusters are reported. The integrated colours have been used for a long time to date the extragalactic clusters (e.g. Girardi & Salaris 2001, and references therein; Hunter et al. 2003). As we have discussed in Section 2, the colour evolution of the entire population of the

Table 3. Observed MS integrated magnitude and colours of galactic clusters obtained in the present work.

Cluster	$(m - M)$ (mag)	$E(B - V)$ (mag)	Age log t	M_V (mag)	$(U - V)_0$ (mag)	$(U - B)_0$ (mag)	$(B - V)_0$ (mag)	$(V - R)_0$ (mag)	$(V - I)_0$ (mag)
Be 20	15.00	0.12	9.70	−0.88	–	–	0.53	0.31	0.63
Be 42	11.30	0.76	9.30	2.82	0.65	0.04	0.62	0.33	–
Be 64	16.20	1.05	9.00	−4.33	0.23	0.01	0.23	0.02	0.13
Be 69	14.30	0.65	9.00	−2.15	0.29	0.07	0.21	0.09	0.29
Be 86	12.60	0.70	7.00	−5.42	−0.96	−0.78	−0.18	0.06	–

Note. The complete table is available in electronic form only (see Supporting Information).

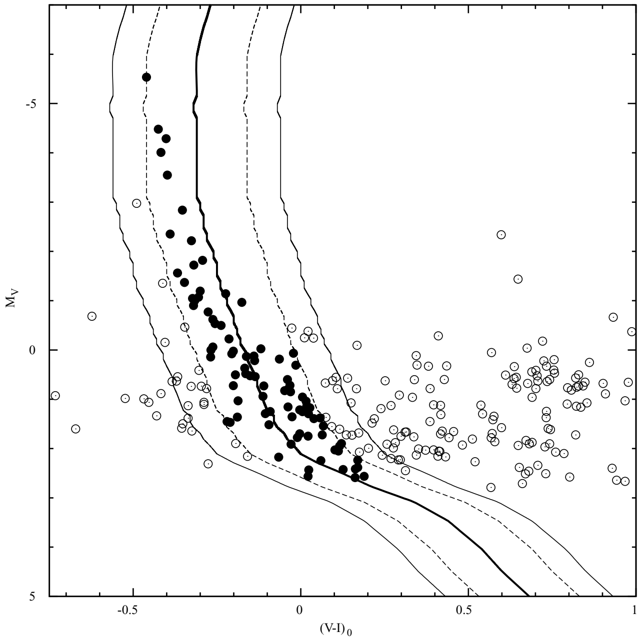

Figure 7. The blue and red envelopes around the MS (thick curve) in the case of LMC 327. The width of the envelope in colour is 0.3 mag (dashed curves) and 0.5 mag (thin curves).

cluster (i.e. MS + giants) is significantly influenced by the stochastic fluctuations in comparison to the colour evolution of the MS population of the cluster, therefore the age of clusters derived from the whole population integrated colours must be subject to a greater uncertainty in comparison to those obtained by using the only MS population sample.

The Optical Gravitational Lensing Experiment (OGLE) has reported *BVI* photometry for 745 LMC clusters (Pietrzynski et al. 1999) and 238 SMC clusters (Pietrzynski et al. 1998). We have used the above-mentioned catalogues to calculate the integrated parameters of the MS population of the LMC and SMC clusters. The selection of a data sample representing the MS of MC clusters is an arduous task. The width of the observed MS depends on the presence of binaries, photometric errors, intracluster reddening and spread in metallicity. The presence of equal mass binaries can redden the distribution by ∼0.1 mag (see Sandhu et al. 2003). Udalski et al. (1998) have reported an error of ∼0.05 mag in the estimation of colours. Since $E(B-V)$ for MC clusters varies from ∼0.05 to ∼0.15, an average $E(B-V)$ value may introduce an error of ∼0.1 mag in dereddened colours. The parameters discussed above can broaden the observed MS by $\sim\pm0.15$ mag. To select a MS sample, we plotted blue and red envelopes, having a width of 0.3 mag, around the MS as shown in Fig. 7. We found that the selected width (0.3 mag) for the MS may exclude about 10–15 per cent of stars towards the brighter end, whereas the blue envelope of the MS excludes about 50 per cent of stars towards the fainter end ($M_V > 1$). An increase in width of the MS up to 0.5 mag in colour includes almost all the brighter stars ($M_V > 1$) in the sample, whereas towards the fainter end the blue envelope includes about 80–90 per cent of stars bluewards of the MS. A further increase in the width of the MS includes an insignificant number of stars towards the bluer side of the MS, but includes more stars towards the red side of the MS. Therefore, we select the width of the MS as 0.5 mag. A broader MS will have a higher probability to include non-MS stars.

The integrated parameters were calculated using a distance modulus of 18.54 and 18.93 mag for the LMC and SMC, respectively (Keller & Wood 2006). For the age range log(age) ≤ 7.3, 7.3 ≤ log(age) ≤ 8.4, and log(age) > 8.4, the reddening $E(B-V)$ is assumed to be 0.14, 0.08 and 0.03 mag for LMC clusters. For the age range log(age) ≤ 7.3, 7.3 ≤ log(age) ≤ 8.4 and log(age) > 8.4, the reddening $E(B-V)$ is assumed to be 0.1, 0.08 and 0.03 mag for SMC clusters. The stars above the turnoff points are not considered for estimating the integrated parameters. All the probable MS stars of the cluster region lying within the width of 0.5 mag were used to calculate the integrated parameters by summing the flux of each star. The catalogues of integrated parameters of LMC and SMC clusters are given in Tables 4 and 5, respectively. A sample of these tables is shown here while the complete catalogues are available in electronic form. The clusters having MS members of fewer than 10 stars have not been included in the catalogue. The

Table 4. Observed MS integrated magnitude and colours of LMC clusters obtained in the present work.

OGLE-ID	Other name	Age (log *t*)	M_V (mag)	$(V-I)_0$ (mag)	$(B-V)_0$ (mag)	*N*
LMC0001	HS81	8.33	−2.71	0.04	–	20
LMC0003	BSDL403	8.70	−1.17	0.11	–	13
LMC0004	H88-85	7.60	−2.60	−0.01	–	25
LMC0005	HS83	8.15	−3.80	−0.06	–	42

Note. *N* is the number of stars used to calculate the integrated parameters. The complete table is available in electronic form only (see Supporting Information).

Table 5. Observed MS integrated magnitude and colours of SMC clusters obtained in the present work.

OGLE-ID	Other name	Age (log *t*)	M_V (mag)	$(V-I)_0$ (mag)	$(B-V)_0$ (mag)	*N*
SMC0002	HW11	8.40	−4.25	−0.01	−0.06	149
SMC0003	L19	9.00	−2.96	0.36	0.22	133
SMC0004	B10	–	−3.88	−0.04	−0.13	126
SMC0005	OGLE	–	−1.01	0.09	0.09	12

Note. *N* is the number of stars used to calculate the integrated parameters. The complete table is available in electronic form only (see Supporting Information).

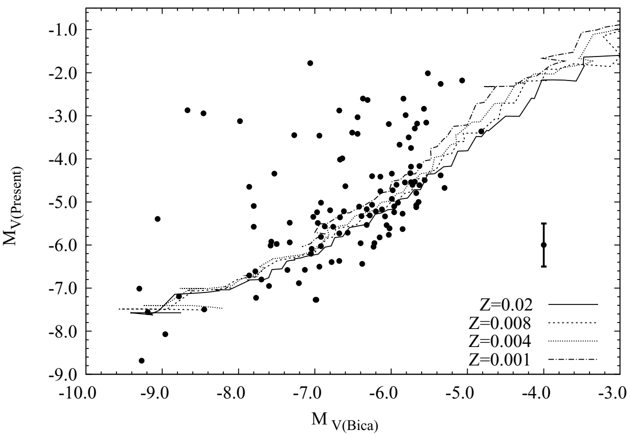

Figure 8. Comparison of observed integrated MS M_V magnitudes of LMC clusters obtained in the present work (MS population) and those given by Bica et al. (1996) for whole cluster population with the present model predictions. The curves represent the model predictions for various metallicities. The typical error in estimation of integrated MS M_V is also shown.

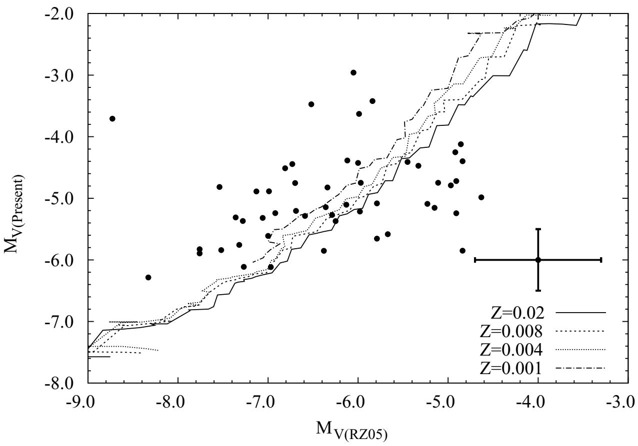

Figure 9. Comparison of observed integrated MS M_V magnitudes of SMC clusters obtained in the present work (MS population) with those given by RZ05 for whole cluster population with the model predictions (continuous curves). The typical errors in estimation of integrated MS M_V and $M_{V(\mathrm{RZ05})}$ are also shown.

ages of the LMC and SMC clusters are taken from Pietrzynski et al. (1999) and Pietrzynski et al. (1998), respectively.

In Fig. 8, we have plotted the magnitude M_V of LMC clusters common in the catalogue of Bica et al. (1992, 1996) and in the present work along with the model predictions obtained in the present work. The apparent V magnitudes by Bica et al. (1992, 1996) are converted to absolute magnitudes by using a distance modulus of 18.54 and $E(B-V) = 0.1$ mag. Fig. 8 shows that the observations are fairly represented by the model predictions. However, a few clusters having $M_V \sim -9.0$ to -7.0 mag (whole population; Bica et al. 1992, 1996) show a large deviation in the sense that M_V estimations for MS populations are too small ($M_V \sim -4.0$ to -1.0 mag). It is noted that most of these clusters are old and have log(age) ≥ 9.0. In the estimation of MS population integrated magnitudes and colours, we have excluded bright evolved stars above the turn-off, where as in the whole population integrated parameters (by Bica et al.), the possibility of bright field star contamination is higher. Moreover, the stochastic fluctuations increase for clusters older than log(age) ~ 9.0. However, a comparison

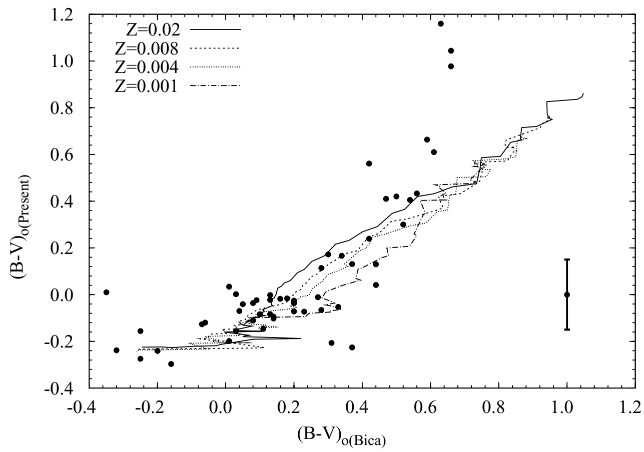

Figure 10. Comparison of observed MS integrated $(B-V)$ colours of LMC clusters obtained in the present work (MS population) and those given by Bica et al. (1996) for whole cluster population with the model predictions (continuous curves). The typical error in estimation of integrated MS $(B-V)$ colours is also shown.

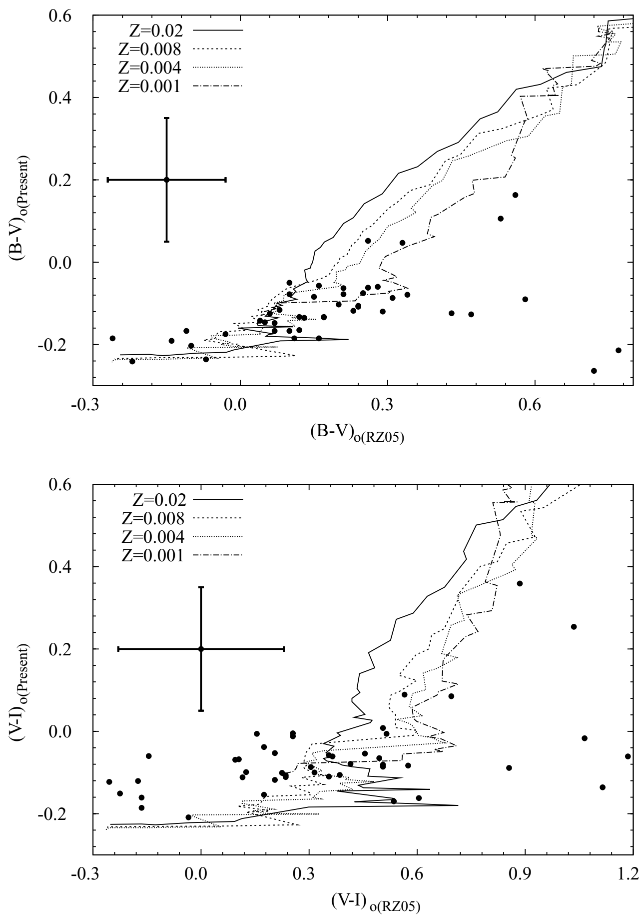

Figure 11. Comparison of observed MS integrated colours of SMC clusters obtained in the present work (MS population) and those given by RZ05 for whole cluster population with the model predictions (continuous curves). The typical errors in estimation of integrated MS and whole population colours are also shown.

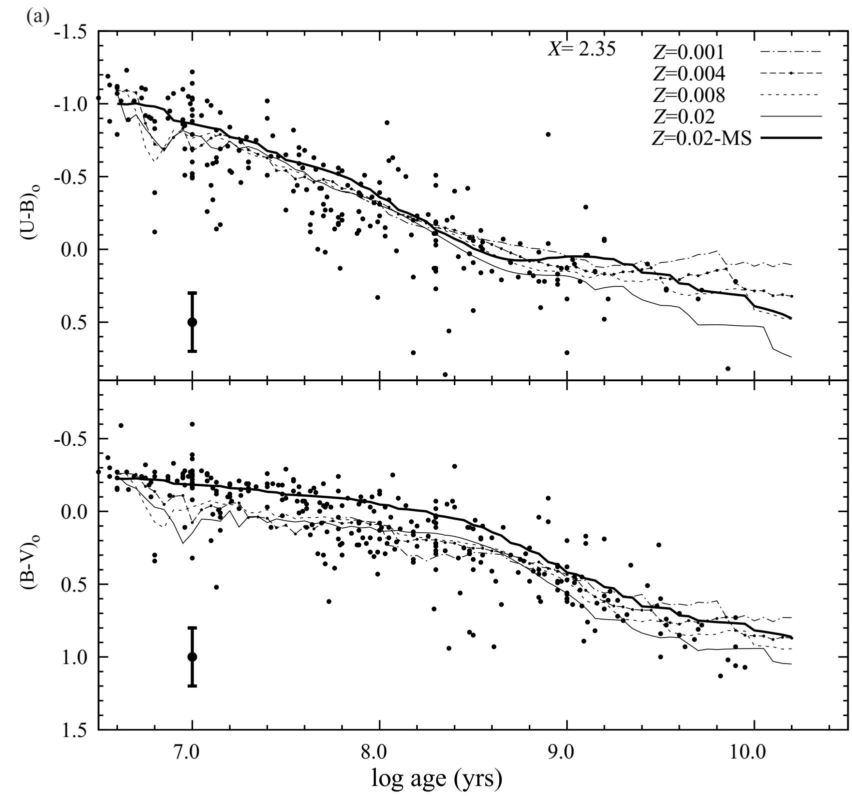

Figure 12. (a) A comparison of present model predictions for whole cluster population (dashed and thin curves) and the MS population (thick curve) with the observational data (whole population) for galactic open clusters. The typical errors in estimation of integrated colours are also shown. (b) Same as Fig. 12(a), but for the MS population only. The typical errors in estimation of integrated colours are also shown.

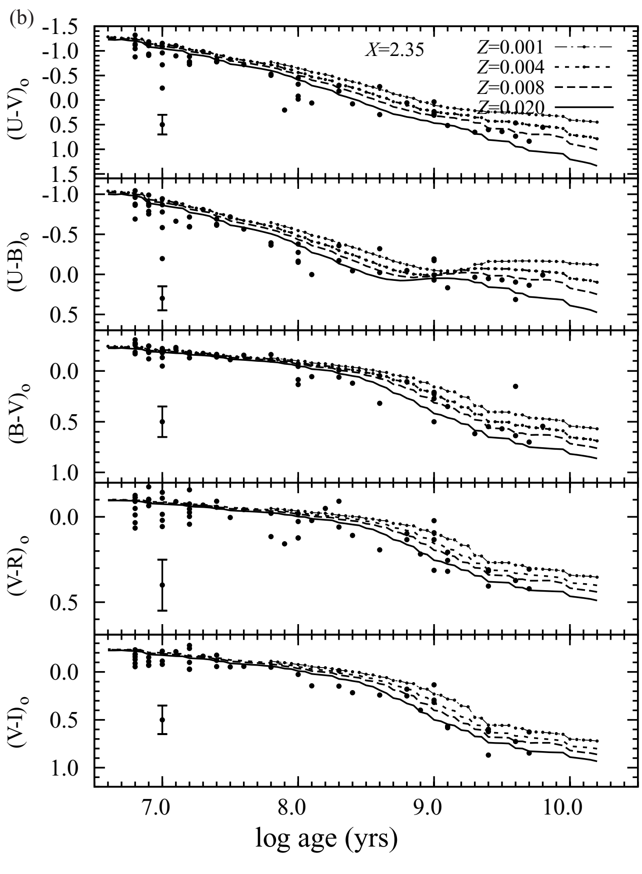

Figure 12 – *continued*

of observed M_V magnitudes of SMC clusters obtained by RZ05 and those obtained in the present work with the model predictions shown in Fig. 9 indicates a rather satisfactory agreement. The apparent V magnitudes by RZ05 are converted to absolute magnitudes by using a distance modulus of 18.93 and $E(B-V) = 0.1$ mag. A comparison of observed colours of LMC and SMC clusters with the model predictions is shown in Figs 10 and 11 which indicates a fair agreement between observations and model predictions. The majority of the observational data points lie within 0.15 mag from the model predictions. This can be considered as the maximum uncertainty in the observed colours, i.e. a combined uncertainty in observed MS colours and whole population colours. If we consider that both the samples have same order of uncertainty, then each sample will have an uncertainty of 0.1 mag.

4 COMPARISON OF THEORETICAL PREDICTIONS WITH THE OBSERVATIONS

The colour of star clusters which form on a short time-scale is an obvious choice to test the theoretical models. The star clusters in the Galaxy and the MC have ages from a few million years to a few billion years. In this section, we compare theoretical predictions obtained for synthetic clusters with the observed integrated parameters of star clusters in the Milky Way and MCs.

4.1 Galactic open clusters

In Fig. 12(a), we compare the evolution of $(U-B)$ and $(B-V)$ colours of a synthetic cluster ($X = 2.35$, $Z = 0.02$) having a MS as well as an evolved population with the observed integrated

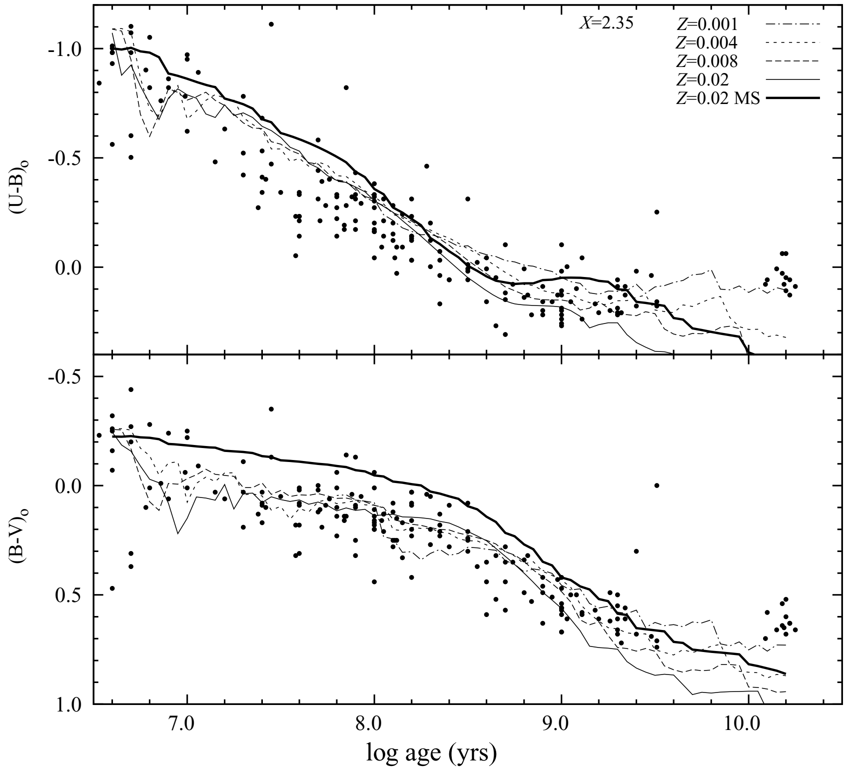

Figure 13. Comparison of the present model (dashed and thin curves: whole population; thick curve: MS population) with the observational data for LMC clusters (whole population) by Bica et al. (1996).

parameters of open clusters. Observed integrated parameters for 319 clusters (whole population) have been taken from Lata et al. (2002). Although the observational data show a large scatter, the agreement between the theoretical and observed evolution of colours is good. Observed integrated $(B - V)$ colours of clusters having log(age) $\leq$7.5 are better explained by the MS model. Fig. 12(b) shows a comparison of integrated colours of the MS population only, which indicates a good agreement between the theoretical and observed colours.

4.2 LMC clusters

The comparison of $(U - B)$ and $(B - V)$ colour evolution by Bica et al. (1996) with the present model is given in Fig. 13. The age of the clusters is taken from Sagar & Pandey (1989) and Mackey & Gilmore (2003). A constant reddening of $E(B - V) = 0.10$ mag is applied to the observed data. The comparison between observed data and the model indicates a fair agreement. A few clusters having log(age) $\leq$ 8.0 are relatively bluer and can be explained by the MS model. Fig. 13 further confirms the well-known fact (cf. Olszewski et al. 1991; Olszewski, Suntzeff & Mateo 1996) that the oldest clusters (age > 10 Gyr) in the LMC are significantly metal poor.

In Fig. 14, integrated $(B - V)$ and $(V - I)$ colour evolution of the MS population obtained in the present work is compared with the colour evolution obtained for synthetic clusters. As can be seen, the MS population colours are not affected by the stochastic fluctuations. The comparison indicates a nice agreement between the observed and synthetic model colour evolution showing that the clusters having age >500 Myr are distributed around the lower metallicity ($Z = 0.004$) model. However, the observed $(V - I)$ colours for clusters having log(age) >8.7 are found to be bluer even than those for models with $Z = 0.004$.

4.3 SMC clusters

RZ05 compared their observational data with the models by Leitherer et al. (1999) and Anders & Fritze-v. Alvensleben (2003) and found a systematic difference between their data and the models in the sense that the observed data are too blue for the bluest colours. A large scatter can be seen in the case of two-colour diagrams (TCDs), namely $(U - B)$ versus $(B - V)$, $(V - I)$ versus $(B - V)$ and $(U - B)$ versus $(V - I)$ diagrams.

In Fig. 15, we compare $(U - B)$, $(B - V)$, $(V - I)$ colour evolution of SMC clusters using the data of RZ05 with the present model. For comparison, we assumed a mean reddening $E(B - V) = 0.1$ mag. Comparison shows that for the age range 6.5 $\leq$ log(age) $\leq$ 8.0, the integrated $(B - V)$ and $(V - I)$ colours follow the MS population colour evolution. The same trend has also been noted in the case of galactic open clusters and LMC clusters. Some of the clusters in the age range log(age) > 7.5 follow the whole cluster population sequence predicted by the present model. The observed $(U - B)$ colour evolution is fairly represented by the present model colour evolution.

Figure 16 shows $(B - V)$ and $(V - I)$ colour evolution of the MS population of SMC star clusters. The models for $Z = 0.001$, 0.004, 0.008 and 0.02 are also shown in the figure. The scatter in the observational data is less just like in the case of open and LMC clusters. The observed colour evolution in the age range log(age) $\leq$ 8.0 is fairly explained by the model having $Z = 0.004$ and 0.008,

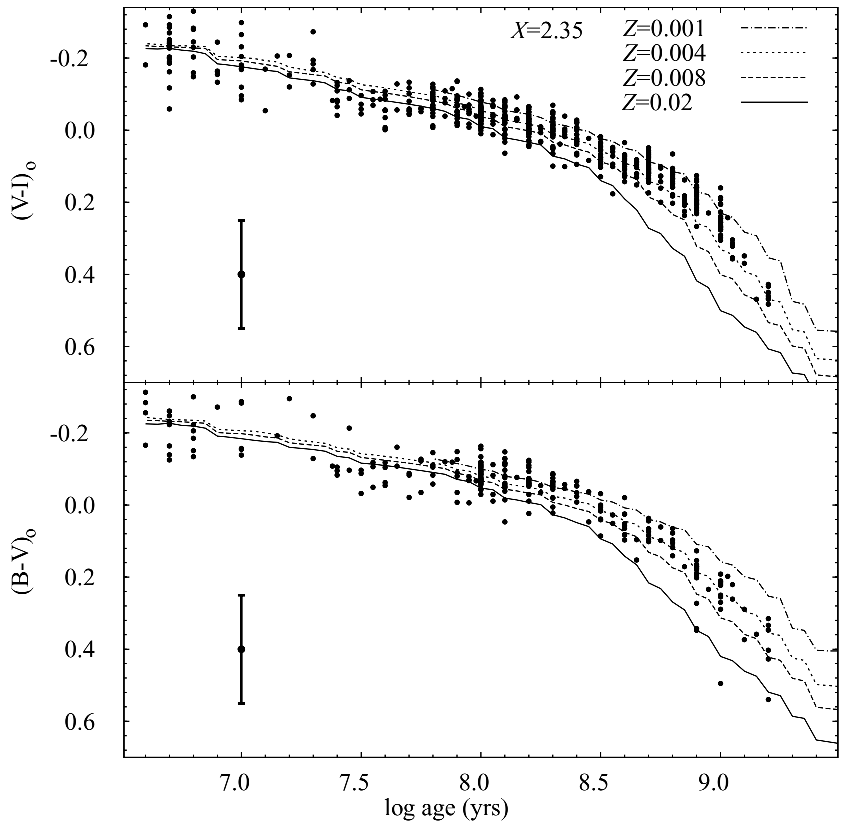

Figure 14. Comparison of observed integrated colours for the MS population of LMC clusters obtained in the present work with the present model predictions. The typical error in estimation of integrated colours is also shown.

whereas clusters having log(age) > 8.0 seem to follow a lower metallicity ($Z = 0.004$) model.

4.4 Effect of stochastic fluctuations on observed MC clusters

Girardi & Bica (1993) have pointed out that the small number of evolved red stars in less populous clusters can cause significant and fast changes in their integrated colours. They pointed out that most of the dispersion in the observed $(U - B)$ versus $(B - V)$ diagram of LMC star clusters (their fig. 3) can be attributed to the stochastic effects, especially for clusters older than ∼50 Myr, for which the internal reddening is expected to be negligible. Girardi et al. (1995) concluded that in a sample with such low-luminosity clusters as that of Bica et al. (1996), the stochastic effects play a significant role in the interpretation of evolution of integrated clusters.

As Girardi & Bica (1993) pointed out that the dispersion in the observed colour–colour and colour evolution diagram is mainly due to stochastic effects, we used Figs 13–16 to study the effect of stochastic fluctuations on the observed $(B - V)$ colours of MC star clusters. Fig. 17 shows standard deviations of mean $(B - V)$ colours in a bin of log(age) = 0.2 as a function of age, which clearly shows that the dispersion in the case of the MS population is significantly less than in the case of the whole cluster population.

4.5 Age–metallicity relation for MC star clusters

The age–metallicity relation in the MC star clusters has been well known for a long time. For example, a compiled catalogue by Sagar & Pandey (1989) yields $0.001 \leq Z \leq 0.01$ with a mean value of $Z \sim 0.005$ for cluster having log(age) ∼ 7.0–9.2 (cf. their fig. 3), whereas the study of LMC clusters by Olszewski et al. (1991) indicates a mean value of $Z \sim 0.008$ with a range of $Z \sim 0.02$–0.004. Bica et al. (1998) have derived mean metallicity for the intermediate age LMC clusters (9.0 ≤ log(age) ≤ 9.4) as $Z \sim 0.005$ and found that the metallicities obtained by them are significantly lower than those reported by Olszewski et al. (1991) for a sample of clusters of similar age, but their values are in good agreement with several contemporary studies. In a recent study, Kerber, Santiago & Brocato (2007) have found that the LMC clusters younger than log(age) ∼ 9.5 have $Z \sim 0.006$ with a considerable scatter. They have also pointed out that the metallicities by Olszewski et al. (1991) are higher as compared to their values and have also discussed possible reasons for such a discrepancy. The above discussions indicate that the colour evolution of MC star clusters discussed in Sections 4.2 and 4.3 is in accordance with the observed age–metallicity relation for MC star clusters. However, a discrepancy in $(V - I)$ colours for LMC clusters having log(age) ≥ 8.7 has been noted in Section 4.2.

4.6 LMC clusters: $(V - I)$ colour discrepancy

Possible reasons for the discrepancy in $(V - I)$ colours of LMC clusters as noted in Section 4.2 may be: (i) reddening correction, (ii) systematic effects in the model predictions towards older ages and (iii) an anomalous reddening law.

The reddening $E(B - V)$ can be estimated relatively accurately for Galactic open clusters, hence reddening corrections for

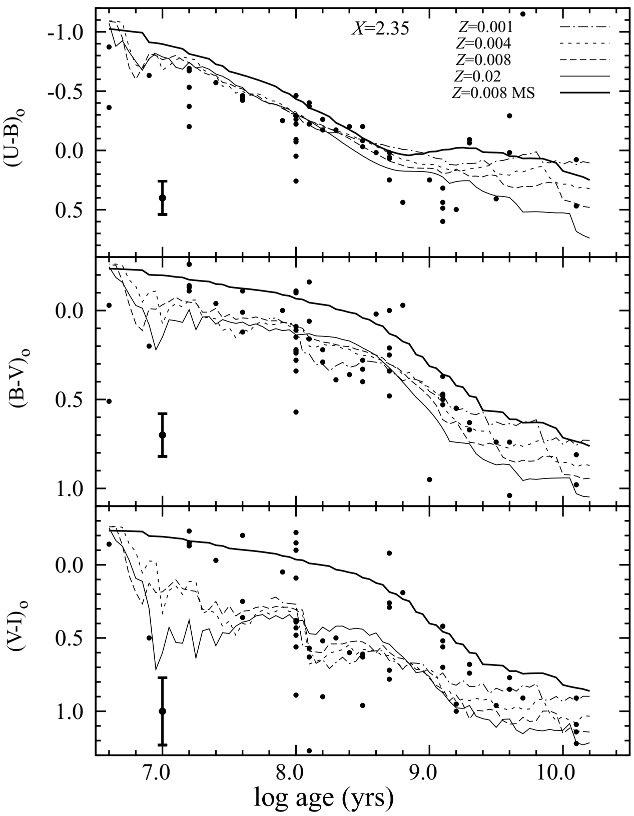

Figure 15. Comparison of observational data by RZ05 with the present model. The dashed and thin curves represent the whole population and the thick curve represents the MS population of the clusters. The typical errors in estimation of integrated colours are also shown.

individual clusters were applied. Fig. 12(b) shows a fair agreement of the observed colour evolution of the MS population of Galactic open clusters with the model predictions having $Z = 0.02$, which suggests that the model predictions do not have any systematic effect. In the case of MC star clusters, we applied mean values of $E(B - V)$ for three age groups (cf. Section 3.2) assuming a normal reddening law. In the case of LMC clusters having log(age) $\geq$ 8.7, a mean value of $E(B - V) = 0.03$ is applied. As discussed above the $(B - V)$ colour evolution of LMC clusters fairly agrees with the model predictions. The above facts indicate that the systematic effects in model predictions and reddening correction should not be the possible reasons for the discrepancy in $(V - I)$ colours.

5 TWO-COLOUR DIAGRAMS

5.1 $(V - I)$ versus $(B - V)$ diagram

Fig. 18(a) shows the integrated $(V - I)_0$ versus $(B - V)_0$ TCD, assuming a normal reddening law, for galactic open clusters, LMC clusters and SMC clusters along with the present model predictions. Fig. 18(a) indicates that the model nicely explains observed colours of galactic open clusters. The MS population colours of LMC and SMC star clusters having $(B - V) > 0.0$ become sytematically red in $(B - V)$ colour or blue in $(V - I)$ colour. Fig. 18(b) shows reddened $(V - I)$ versus $(B - V)$ TCDs along with model predictions for $Z = 0.008$ for MC star clusters, which also shows a discrepancy in the colours.

This anomaly may be due to an anomalous reddening law in the MCs. The $(B - V)$ versus $(V - I)$ diagram can be used to study the reddening law (cf. Pandey et al. 2003). A least-square fit to the observed LMC and SMC data shown in Fig. 18(b) gives a slope $m_{\rm MC} = 0.9 \pm 0.04$, whereas a fit to the model ($Z = 0.02$ and 0.008) yields a slope $m_{\rm normal} = 1.08 \pm 0.01$ and $m_{\rm normal} = 1.12 \pm 0.01$,

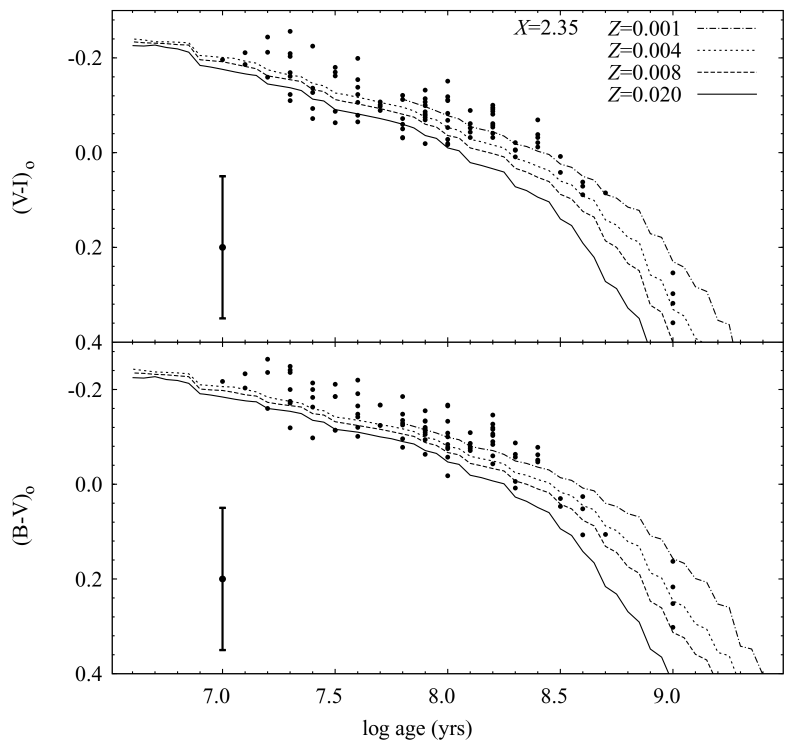

Figure 16. Comparison of observed integrated colours for the MS population of SMC clusters with the present model predictions. The typical error in estimation of integrated colours is also shown.

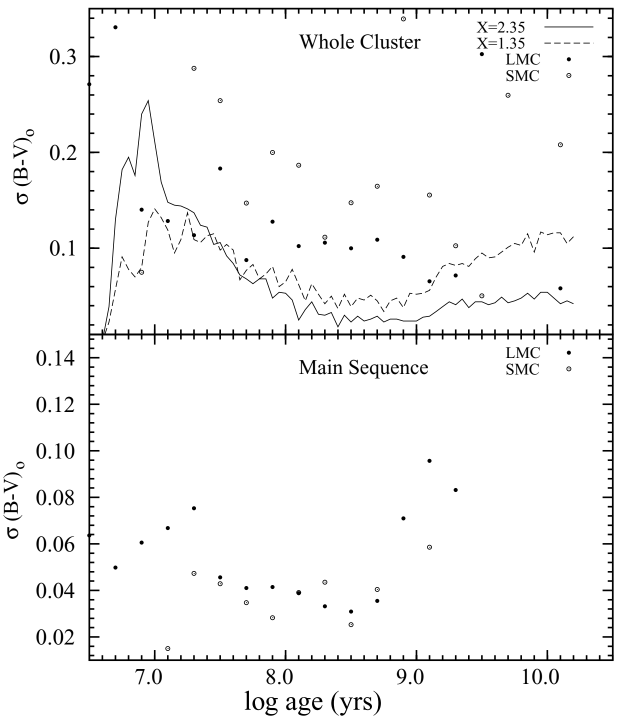

Figure 17. Effect of stochastic fluctuations on observed colours of MC star clusters (filled circles: LMC clusters; open circles: SMC clusters). The continuous and dashed curves show the effect of stochastic fluctuations on colours of simulated clusters having a post-MS population.

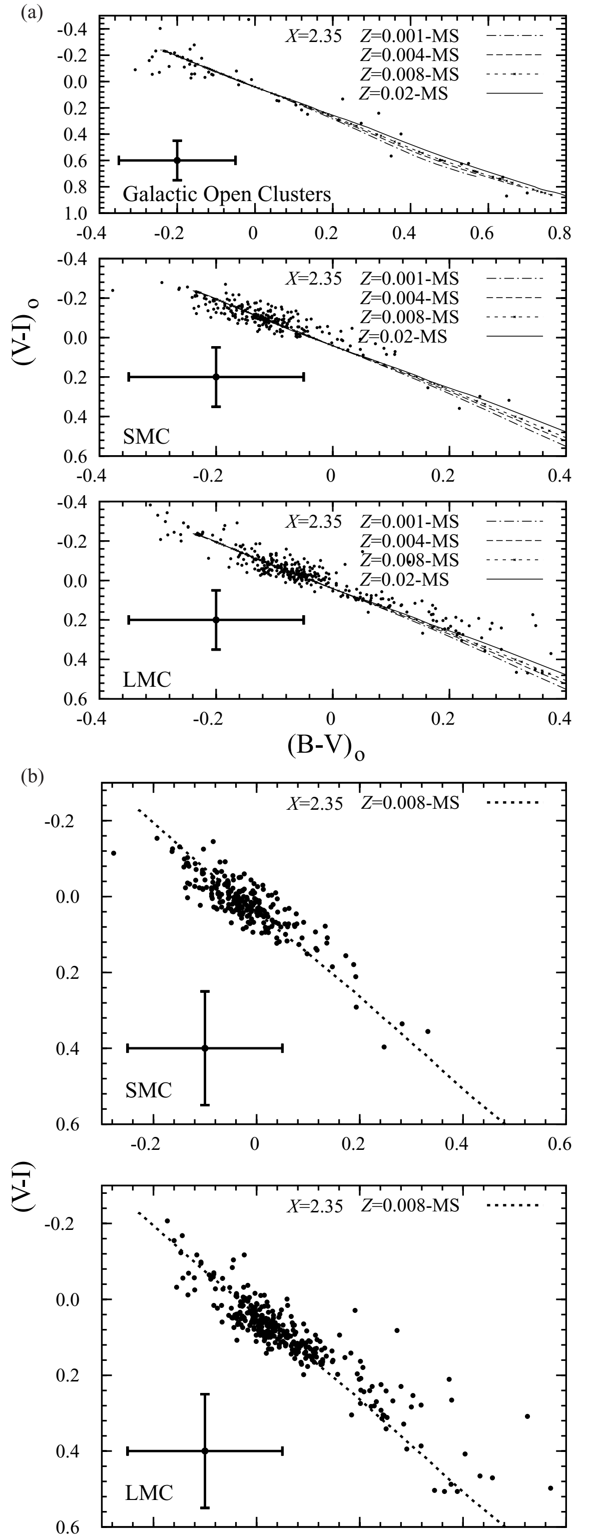

Figure 18. (a) $(B - V)_0$ versus $(V - I)_0$ TCD for the MS population of galactic open clusters (upper panel), SMC clusters (middle panel) and LMC clusters (lower panel) compared with the present model. The typical errors in estimation of integrated colours are also shown. (b) $(B - V)$ versus $(V - I)$ TCD for the MS population SMC clusters (upper panel) and LMC clusters (lower panel) compared with the present model with $Z = 0.008$. The typical errors in estimation of integrated colours are also shown.

respectively. Adopting the procedure described by Pandey et al. (2003), the value of total to selective absorption $R_{V(\mathrm{MC})}$ towards the MCs can be obtained as follows:

$$R_{V(\mathrm{MC})} \simeq \frac{m_{\mathrm{MC}}}{m_{\mathrm{normal}}} \times R_V,$$

assuming $R_V = 3.1$, the value of $R_{V(\mathrm{MC})}$ comes out to be 2.6–2.5 $\pm$ 0.1 indicating a lower grain size towards the MC star clusters.

There is evidence for the anomalous reddening law in the MCs. For the SMC bar, Gordon & Clayton (1998) and Gordon et al. (2003) have found $R_V = 2.7 \pm 0.1$, which is consistent with the value 2.7 $\pm$ 0.2 reported by Bouchet et al. (1985). A smaller value of $R_V = 2.76 \pm 0.09$ was reported for the LMC2 super-shell sample by Gordon et al. (2003). However, for the LMC average sample Gordon et al. (2003) have found $R_V = 3.41 \pm 0.06$.

The metallicity in the MCs is substantially lower than in the Milky Way and there are indications that the measured extinction curves towards the LMC and SMC differ from typical extinction curves in the Milky Way (cf. Weingartner & Draine 2001). Because of lower metallicity, the typical molecular clouds in the LMC and SMC are bigger but more diffuse than those in the Milky Way (Pak et al. 1998). Therefore, dust grains in the LMC and SMC may not spend as much time in dark, shielded environments as dust grains in the case of the Milky Way. This may result in small size dust grains, which consequently yield a low value of R_V in the in the LMC and SMC.

A comparison of RZ05 data with the present model (Fig. 19) indicates that majority of the observed data are fairly explained by the MS population, however, some of the observations follow the whole cluster model. A comparison of Figs 18(a) and 19 also indicates that the scatter in MS population data is significantly less than the data of RZ05.

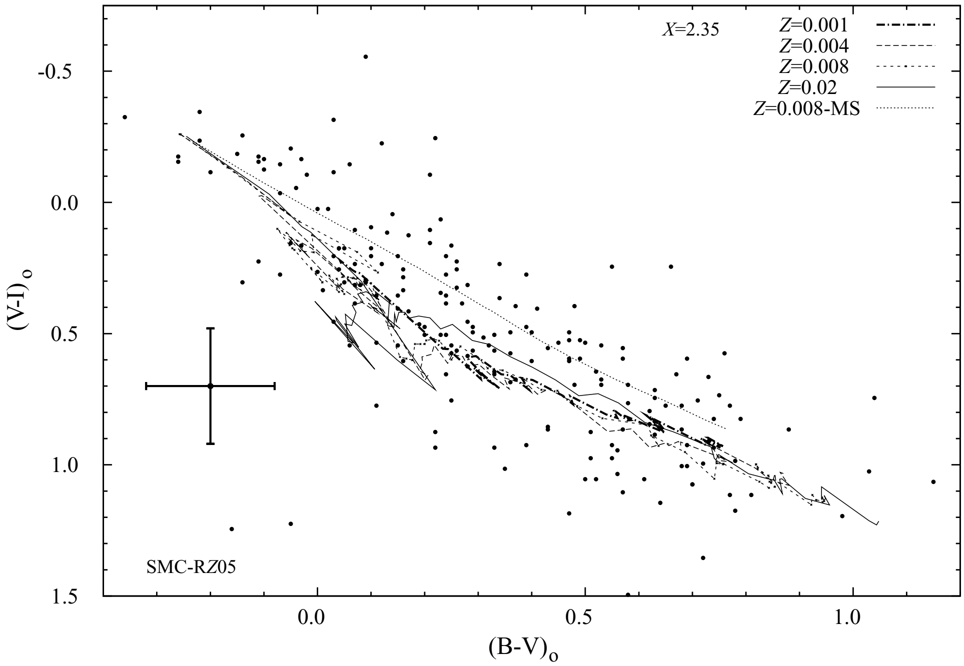

Figure 19. Integrated colours of SMC clusters by RZ05 are compared with the present model (dashed and thin curves: whole population; thick curve: MS population). The typical errors in estimation of integrated colours are also shown.

5.2 $(U - B)$ versus $(B - V)$ diagram

Fig. 20 shows the $(U - B)_0$ versus $(B - V)_0$ TCD for the MS population ($Z = 0.02$, $X = -2.35$) and whole cluster population of a synthetic cluster ($Z = 0.004, 0.008, 0.02$, $X = -2.35$) and compares it with the observational data of galactic open clusters, LMC and SMC clusters. In the case of LMC data (Bica et al. 1992, 1996) and SMC data (RZ05), a mean reddening of $E(B - V) = 0.1$ mag has been assumed.

Fig. 20 shows a large scatter in the observational data. The amount of scatter is almost the same in various sets of data. In all three samples, a large number of the bluest clusters ($(B - V)_o < 0.0$) follow the MS population relation. Note that a similar trend in the age range $6.5 \leq \log(\text{age}) \leq 8.0$ has been observed while studying the $(B - V)$ colour evolution (cf. Section 4). The remaining clusters show a large scatter around the theoretical $(U - B)$ versus $(B - V)$ relation for the whole cluster.

6 CONCLUSION

In this paper, we present integrated magnitudes and colours for synthetic clusters using the synthetic CMDs of star clusters. The integrated parameters have been obtained for the whole cluster population as well as for the MS population of star clusters. We have also estimated observed integrated magnitudes and colours of the MS population of galactic open clusters, LMC and SMC star clusters. The relation between observed integrated colours for the whole cluster population and MS population are fairly explained by the model predictions obtained in the present work. This indicates that the estimated observed integrated colours of the MS population of MC clusters fairly represent the MS population of the MC clusters. The main conclusions of the present study are as follows.

(i) The present model suggests that colour evolution of the MS population of star clusters is not affected by the stochastic

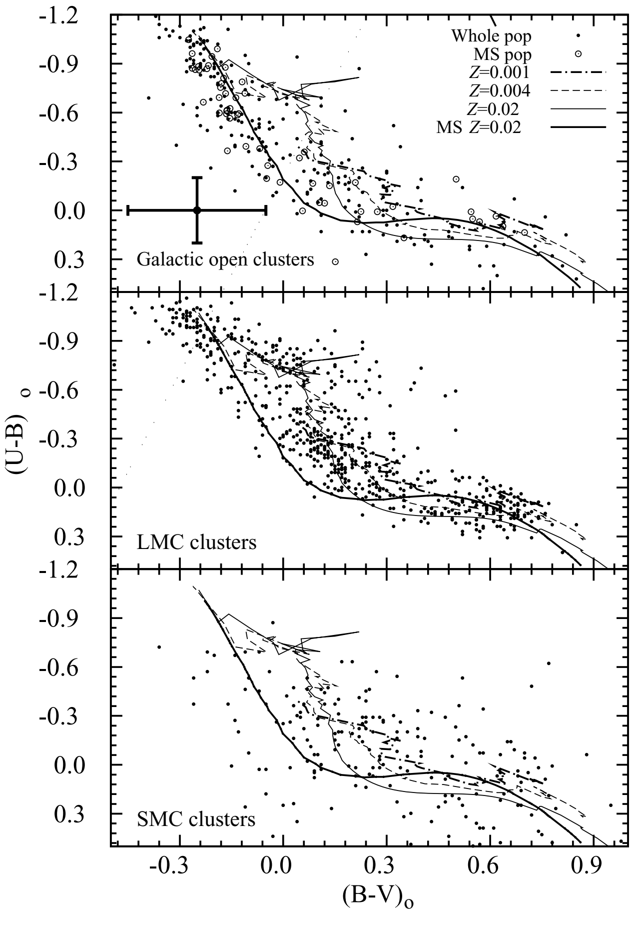

Figure 20. $(U - B)_0$ versus $(B - V)_0$ synthetic colour–colour diagram compared with the observational data of Galactic open clusters (whole population: Pandey et al. 1989; Lata et al. 2002, MS population: present work), LMC (Bica et al. 1996) and SMC (RZ05) clusters.

fluctuations. Stochastic fluctuations significantly affect the colour evolution of the whole cluster population. The fluctuations are maximum in $(V - I)$ colour in the age range 6.7 < log(age) < 7.5. The observed data of MC star clusters also indicate that the effect of stochastic fluctuations on estimation of integrated colours of the MS population is significantly less than in the case of colours of the whole population.

(ii) The evolution of the integrated magnitude of star clusters with age depends on the IMF of the cluster. The presence of massive stars, i.e. shallow IMF, makes the integrated magnitude of the cluster brighter and fade relatively faster than the clusters having a steeper IMF. The variation of the IMF has an insignificant effect on the colour evolution of star clusters after log(age) $\sim$ 7.5. However, the colour evolution in the age range 6.7 < log(age) < 7.5 is significantly governed by the choice of the IMF. This further confirms the earlier results (e.g. by Chiosi et al. 1988; Pandey et al. 1989; Girardi et al. 1995; Bruzual & Charlot 2003, and references therein).

(iii) The metallicity variation does not show any significant effect on the evolution of magnitude as well as on the colours of clusters having log(age) $\leq$ 8.5. For older clusters colours become bluer with the decrease in metallicity. This is in accordance with the results obtained in earlier studies (e.g. Girardi et al. 1995).

(iv) The $(U - B)$ and $(U - V)$ colours for whole cluster population are slightly bluer in comparison to those reported by Brocato et al. (1999) and Maraston (1998).

(v) The $(B - V)$ colour evolution for the whole cluster population is in agreement with those reported by Brocato et al. (1999) and Maraston (1998). The $(V - I)$ colour evolution for log(age) $\geq$ 7.0 is in reasonable agreement with that given by Brocato et al. (1999).

(vi) The $(V - R)$ colour evolution obtained in the present work is in good agreement with that given by Brocato et al. (1999), whereas the $(V - R)$ colour evolution reported by Maraston (1998) does not agree with the present work as well as with that given by Brocato et al. (1999).

(vii) Evolution of integrated colours of the MS population of the clusters in the Milky Way, LMC and SMC obtained in the present study is nicely explained by the present synthetic cluster model. A comparison of the present model with the observational data indicates that the MC star clusters having age $\geq$ 500 Myr seem to favour a metallicity lower than $Z = 0.008$. Observed integrated colours of the whole population of Milky Way, LMC and SMC star clusters are also explained fairly well by the present model.

(viii) The $(V - I)$ versus $(B - V)$ TCD for the MS population of the Milky Way star clusters shows a fair agreement between the observations and present model. However, the diagrams for the LMC and SMC indicate a discrepancy in colours. An anomalous reddening law towards the MC may be a possible reason for the discrepancy.

(ix) Comparison of the synthetic $(U - B)$ versus $(B - V)$ relation with the observed data (i.e. whole cluster population) of Milky Way, LMC and SMC star clusters indicates that the majority of the bluest clusters $[(B - V)_0 < 0.0]$ follow the MS population relation. The observed colour evolution of young clusters [6.5 $\leq$ log(age) $\leq$ 8.0] in the Milky Way, LMC and SMC also indicates that a large number of young clusters follow the MS population relation.

(x) The $(U - B)$ versus $(B - V)$ colour–colour diagram and colour evolution of star clusters are frequently being used to date the clusters by comparing observed data with the model prediction for the whole cluster population. The present results indicate that the dating of the clusters may be erroneous if a proper synthetic model (e.g. whole population model is used without proper statistical techniques to account for the stochastic fluctuations, see Bruzual 2009) is not used.

ACKNOWLEDGMENTS

We are thankful to the anonymous referee for the critical comments which improved the scientific content and presentation of the paper.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

Table 1. Integrated magnitude and colours of synthetic clusters (MS population) obtained in the present work.
Table 2. Integrated magnitude and colours of synthetic clusters (whole population) obtained in the present work.
Table 3. Observed MS integrated magnitude and colours of galactic clusters obtained in the present work.
Table 4. Observed MS integrated magnitude and colours of LMC clusters obtained in the present work.
Table 5. Observed MS integrated magnitude and colours of SMC clusters obtained in the present work.

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