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# Photometric Study of the Eclipsing Binary System AR Lacertae

by

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ABSTRACT

The geometrical elements of the eclipsing binary system AR Lacertae have been obtained by Fourier analysis of the light changes in the frequency-domain which was developed by Kopal (1975 a, b, c, d, e) and compared with our earlier (Srivastava, 1981) results obtained by employing Russell and Merrills's (1952) method. The absolute dimensions have been obtained using the spectroscopic elements given by Sanford (1951), and the new geometrical elements. The primary component lies fairly on the Main Sequence while the secondary component falls above the Main Sequence. The Roche constants indicate that the system seems to be a detached one apparently. However, evolutionary discussion shows that the possibility of AR Lac being in a semi-detached state of evolution can not be ruled out.

## 1. Introduction

The system AR Lac ($= \mathrm{BD}+45°3813 = \mathrm{HD}\ 210334 = \mathrm{HV}2980$) was found to be a variable by Miss Leavitt (1907), and its light variability was later confirmed by Wendell (1907). Jacchia (1929) and Loreta (1929, 1930) established it to be an eclipsing binary. Parenago (1930, 1938), Schneller and Plaut (1932), Zverev (1936) and Himpel (1936) also confirmed the light variation of the system. Harper (1933) presented the spectrographic elements. Wyse (1934) classified the spectra of the system respectively as G5 and K0. He noted that the K0 component had sharp H and K emission lines. Wood (1946) presented the elements of the system and called attention to the possibility of intrinsic light variation in one of the components. Kron (1947) obtained photoelectric observations of the system, and noting irregularities during the partial phases, proposed an explanation based on the presence of "star spots" in the photosphere of the primary (G5) component. Sanford (1951) presented the spectroscopic elements of AR Lac. Struve (1952) discussed the spectra of the system in the light of "turbulent spot" hypothesis. Popper (1967) listed the masses of the components. Catalano and Rodonó (1967) confirmed luminosity variation and irregular period changes. They observed persistent (nearly sinusoidal) wave-like distortion in the light-curve outside eclipse which migrates slowly towards decreasing orbital phase.

Oliver (1974) and Hall (1976) classified AR Lac as a member of RS CVn group. Chambliss (1976) discussed three colour observations of the system and found light variations of the order of $0\overset{m}{.}04$. Hall *et al.* (1976) examined 18 light curves, obtained between 1926 and 1974, confirming that there is a wave-like distortion outside eclipse.

Recently, Karle *et al.* (1977) have presented light minima observations of the system. Naftilan and Drake (1977) have presented spectroscopic study of the system while Rhombs and Fix (1977) and Weiler (1978) have presented its spectrophotometry. Eggen (1978) presented colours of the system and Scarfe and Barlow (1978) gave light minima. Hoffmann (1980) presented photoelectric and spectrographic observations of AR Lac but his observations did not confirm the vave-like distortion around phase 0.72 as suggested by Hall *et al.* (1976). Zeilik *et al.* (1980) presented *UBVR* and *JHKL* photometry of selected RS CVn stars including AR Lac.

Very recently, Kurutaç *et al.* (1981), Ertan *et al.* (1982) and Nha and Kang (1982) have presented two colour photometry of the system. Naftilan and Aikman (1981) and Kiziloğlu *et al.* (1983) have presented spectroscopic observations of AR Lac.

Many observers (Hjellming and Blankenship, 1973; Gibson and Hjellming, 1974; Owen and Spangler, 1977; Spangler *et al.*, 1977; Woodsworth and Hughes, 1977; Feldman, 1978; Owen and Gibson, 1978, Walter *et al.*, 1978) have presented radio observations of AR Lac.

## 2. Observations

Since the system possesses nearly integral period ($\sim 1\overset{d}{.}98$), it is difficult to secure the full light curve in one observing season. Good photometry of the system was not available in the literature. Earlier observations by Wood (1946) and by Chambliss (1976), who presented photometry respectively in *V* and *UBV* colours, were far from satisfactory, and they had themselves emphasized the need for further observations of the system. Also Koch *et al.* (1970) had classified Wood's (1946) analysis as one carrying poor observational base and had assigned it a low weight. Moreover, the light curve of the system is found to be changing. It was, therefore, considered necessary to obtain fresh observations. Also for a better understanding of period variations, recent times of primary minima were needed. For these reasons, the system was put on our observing programme, and the observations were secured and analysed using the R-M method (Srivastava, 1981).

A total of 16 nights of observations were secured during the period October 1975 to January 1976. The comparison star BD +44°4044 was employed finally to obtain differential magnitudes. The observations have been published (*vide* Srivastava, 1981).

## 3. Epoch and period

During the course of our observations three primary and three secondary minima have been observed, which are reproduced here in Table 1 alongwith the derived values of $O-C$, based on the ephemeris given by Guarnieri *et al.* (1975), viz.:

$$\text{Primary Minimum} = \text{JD } 2439376.4928 + 1^{\mathrm{d}}.983200\, E.$$

Table 1

Minima of AR Lacertae

| Sl. No. | Epoch of minima JD (Hel) | Period | $O-C$ based on Guarnieri *et al.* (1975) epoch (and period) |
|---|---|---|---|
| *Primary minima*: | | | |
| 1. | 2442700.304 ($\pm 0^{\mathrm{d}}.001$) | $1^{\mathrm{d}}.983181$ | $-0^{\mathrm{d}}.032$ |
| 2. | 2442716.199 ($\pm 0^{\mathrm{d}}.001$) | $1^{\mathrm{d}}.983198$ | $-0^{\mathrm{d}}.002$ |
| 3. | 2442730.095 ($\pm 0^{\mathrm{d}}.001$) | $1^{\mathrm{d}}.983207$ | $+0^{\mathrm{d}}.011$ |
| Mean | | $1^{\mathrm{d}}.983195$ | $-0^{\mathrm{d}}.008$ |
| *Secondary minima*: | | | |
| 1. | 2442701.314 ($\pm 0^{\mathrm{d}}.001$) | $1^{\mathrm{d}}.983192$ | $-0^{\mathrm{d}}.014$ |
| 2. | 2442715.167 ($\pm 0^{\mathrm{d}}.001$) | $1^{\mathrm{d}}.983174$ | $-0^{\mathrm{d}}.043$* |
| 3. | 2442717.187 ($\pm 0^{\mathrm{d}}.001$) | $1^{\mathrm{d}}.983196$ | $-0^{\mathrm{d}}.006$ |
| Mean | | $1^{\mathrm{d}}.983187$ | $-0^{\mathrm{d}}.021$ |

If we ignore this value of $O-C$, the average shift from both sets of minima comes out to be of the order of $-0^{\mathrm{d}}.01$, and the mean period based on the three primary minima and the two remaining secondary minima comes out to be $1^{\mathrm{d}}.983195$ ($\pm 0^{\mathrm{d}}.000010$), which is nearly the same as given by Guarnieri *et al.* (1975).

Many authors including, Alexandrovich (1959), Plavéc *et al.* (1961), Karle (1962), Schneller (1962), Dueball and Lehmann (1965), Blanco and Catalano (1970), Hall (1972), Biermann and Hall (1973), Hall *et al.* (1976), De Campli and Baliunas (1979), and Hall and Kreiner (1980), Kurutaç *et. al.* (1981), Ertan *et al.* (1982) and Nha and Kang (1982) have devoted their efforts for the observations of primary minimum and/or period studies of the system.

Theokas (1977) pointed out that the elements by Rügemer (1931), D 2426624.378 $+1\overset{d}{.}983244\ E$, held good at least up to October 1931. Wood (1946) reported an abrupt change in period occuring shortly after Dugan and Wright (1939) completed their survey. Plavéc *et al.* (1961) indicated that erratic or semi-regular changes may be present in the system. Theokas (1977) analysed the observations of Kron made from 1938 to 1948 and found a period decrease of the order of magnitude $10^{-5}$.

## 4. Determination of elements

The elements of the system AR Lac have been derived employing the frequency-domain method of Kopal (1975a, b, c, d, e). In view of the complications present in the light curve, the light curves were smoothened through the normal points of the $U$, $B$ and R observations. The intensities at one degree interval have been read out from these curves and are listed in Table 5 and the moments, $A_2$, $A_4$ and $A_6$ ,of the individual light curves have been obtained by plotting the intensities of the individual light curves against $\sin^2\theta$, $\sin^4\theta$ and $\sin^6\theta$.

The moments, $A_{2m}s$, are represented by the formula:

$$A_{2m} = \int_0^{\sin^{2m}\theta_1} (1-l)\,d(\sin^{2m}\theta), \qquad (m = 1, 2, 3),$$

where $\theta_1$ denotes the angle of first contact of the eclipse and $l$ is the fractional light of the system expressed in terms of maximum light between minima taken as unit, and $\theta$ is the phase angle.

The value of the moment $A_0$ has been obtained from intensity versus phase ($l$, $\theta$) curves of each filter. The values of the moments for the primary eclipse are given in Table 2.

Table 2

Moments of AR Lacertae

| Moments | $U$ filter | $B$ filter | $V$ filter |
|---|---|---|---|
| $A_0$ | $0.555 \pm 0.005$ | $0.460 \pm 0.005$ | $0.415 \pm 0.005$ |
| $A_2$ | $0.0532 \pm 0.0002$ | $0.0421 \pm 0.0002$ | $0.0369 \pm 0.0001$ |
| $A_4$ | $0.00664 \pm 0.00003$ | $0.00488 \pm 0.00003$ | $0.00411 \pm 0.00002$ |
| $A_6$ | $0.000974 \pm 0.000005$ | $0.000666 \pm 0.000006$ | $0.000527 \pm 0.000004$ |

The errors in the integration of the curve $l(\sin^{2m}\theta)$ or in the areas under the curve, or in the moments ($A_{2m}$s) are given by the following formula (vide Al-Naimiy, 1977):

$$\varepsilon = \frac{1}{\sqrt{n}}\left\{\frac{1}{n}\sum_{i=1}^{n}[l_i - f(l_i)]^2\right\}^{\frac{1}{2}}(x_n - x_1),$$

where $\varepsilon$ is the error in $A_{2m}$, $n$ is the number of observed points, $l_i - f(l_i)$ is the distance between the observed point and the light curve, $x_n = \sin^{2m}\theta_n$, for the last observed point, $x_1 = \sin^{2m}\theta_1$, for the first observed point. The error of the moment $A_0$ has been obtained from $l(\theta)$ plot.

As stated earlier, the value of the moment $A_0$ has been obtained from the intensity versus phase angle ($\theta$) plot, and is denoted by $L_1 = 1 - \lambda$, $\lambda$ being the observed fractional luminosity during totality.

Then using the relations:

$$A_2 = L_1\bar{C}_3,$$

$$A_4 = L_1(\bar{C}_3^2 + \bar{C}_2^2),$$

$$A_6 = L_1(\bar{C}_3^3 + 3\bar{C}_2^2\bar{C}_3 + \bar{C}_1\bar{C}_2^2),$$

the values of $\bar{C}_1$, $\bar{C}_2$ and $\bar{C}_3$ have been obtained, which are valid for uniformly bright discs. In order to take into account the limb darkening, a suitable value of the limb-darkening coefficient, $u_1$, has been adopted and, using the relations:

$$\bar{C}_3 = C_3$$

$$\bar{C}_2 = \left(\frac{15 - 7u_1}{5(3 - u_1)}\right)^{1/2} C_2,$$

$$\bar{C}_1 = \frac{3(35 - 19u_1)}{7(15 - 7u_1)} C_1,$$

the values of $C_1$, $C_2$ and $C_3$ have been derived.

From these values the geometrical elements of the system have been derived applying the relations:

$$r_{1,2}^2 = \frac{C_{1,2}^2}{(1 - C_3)C_1 + C_2^2}, \quad \sin^2 i = \frac{C_1}{(1 - C_3)C_1 + C_2^2}.$$

The geometrical elements derived from Kopal's method have been given in Table 3 along with the elements derived earlier employing the method of Russell and Merrill (1952).

Table 3

Geometrical elements of AR Lac

| Elements | Russell and Merrill's method | Kopal's method |
|---|---|---|
| $x$ (assumed) | 0.8 | 0.8 |
| $k\left(=\frac{r_s}{r_g}\right)$ | 0.50 | 0.49 (±0.00) |
| $r_1$ | 0.169 | 0.160 (±0.000) |
| $r_2$ | 0.336 | 0.326 (±0.001) |
| $i$ | 76°2 | 83°5 (±0.0) |

The errors of determination of the elements have been obtained from errors involved in the values of moments, and are given in Table 3, using the relations (*vide* Kopal, 1982):

$$A_0\delta\bar{C}_3 = \delta A_2-\bar{C}_3\delta A_0,$$

$$A_0\delta\bar{C}_2^2 = \delta A_4-2\bar{C}_3\delta A_2+(\bar{C}_3^2-\bar{C}_2^2)\,\delta A_0,$$

$$A_0\bar{C}_2^2\delta\bar{C}_1 = \delta A_6-(3\bar{C}_3+\bar{C}_1)\,\delta A_4+(3\bar{C}_3^2+2\bar{C}_1\bar{C}_3-3\bar{C}_2^2)\,\delta A_2-$$
$$-\bar{C}_3(\bar{C}_3^2+\bar{C}_1\bar{C}_3-3\bar{C}_2^2)\,\delta A_0.$$

From the above relations the values of $\delta\bar{C}_1$, $\delta\bar{C}_2$ and $\delta\bar{C}_3$ have been obtained, and applying to these the value of limb-darkening coefficient, the values of $C_1$, $C_2$ and $C_3$ have been derived. From these values, the errors of the orbital elements have been evaluated using the relations:

$$\delta(\cot^2 i) = 2\left(\frac{C_2}{C_1}\right)\delta C_2-\left(\frac{C_2}{C_1}\right)^2\delta C_1-\delta C_3,$$

$$\frac{\delta r_1^2}{r_1^2} = \frac{\delta C_1}{C_1}-\sin^2 i\cdot\delta(\cot^2 i),$$

$$\frac{\delta r_2^2}{r_2^2} = 2\frac{\delta C_2}{C_2}-\frac{\delta C_1}{C_1}-\sin^2 i\cdot\delta(\cot^2 i).$$

In deriving the above elements, the values of $1-\lambda_1$, $1-\lambda_2$, $L_1$, $L_2$ and $a_0$ ($=1$) have been taken to be the same as published earlier (*vide* Srivastava, 1981). The subscripts 1 and 2 refer to the primary and the secondary components, respectively in the above table, while the subscripts $s$ and $g$ refer to smaller and the greater star, respectively.

## 5. Absolute dimensions

The absolute dimensions have been obtained using the spectroscopic elements given by Sanford (1951), as were used earlier by the author (Srivastava, 1981) and are listed in Table 4 along with the absolute dim-

Table 4

Absolute dimensions of AR Lacertae

| Elements | Russell and Merrill's method | Kopal's method |
|---|---|---|
| $A\,(R_{\odot})$ | 6.51 | 6.35 |
| $m_1\,(\odot)$ | 1.41 | 1.32 |
| $m_2\,(\odot)$ | 1.42 | 1.33 |
| $R_1\,(\odot)$ | 1.10 | 1.02 |
| $R_2\,(\odot)$ | 2.19 | 2.07 |
| $\varrho_1\,(\odot)$ | 0.256 | 0.297 |
| $\varrho_2\,(\odot)$ | 0.033 | 0.036 |
| $M_1\,(\mathrm{bol})$ | $4\overset{m}{.}19$ | $4\overset{m}{.}35$ |
| $M_2\,(\mathrm{bol})$ | $3\overset{m}{.}41$ | $3\overset{m}{.}53$ |
| $M_1\,(\mathrm{vis})$ | $4\overset{m}{.}51$ | $4\overset{m}{.}67$ |
| $M_2\,(\mathrm{vis})$ | $3\overset{m}{.}87$ | $3\overset{m}{.}99$ |

Table 5

Observations of AR Lacertae

| Phase in degrees ($\theta$) | Intensity (in terms of the intensity outside eclipse as unity) | | |
|---|---|---|---|
| | U filter | B filter | V filter |
| 0.0 | 0.445 | 0.540 | 0.585 |
| 1.0 | .445 | .540 | .565 |
| 2.0 | .445 | .540 | .585 |
| 3.0 | .445 | .540 | .585 |
| 4.0 | .445 | .540 | .585 |
| 5.0 | .445 | .540 | .585 |
| 6.0 | .445 | .540 | .585 |
| 7.0 | .447 | .540 | .585 |
| 8.0 | .454 | .540 | .590 |
| 9.0 | .467 | .545 | .602 |
| 10.0 | .487 | .565 | .617 |
| 11.0 | .517 | .582 | .640 |
| 12.0 | .550 | .620 | .663 |
| 13.0 | .583 | .653 | .690 |
| 14.0 | .615 | .687 | .715 |
| 15.0 | .647 | .695 | .742 |
| 16.0 | .683 | .750 | .770 |
| 17.0 | .717 | .780 | .800 |
| 18.0 | .750 | .813 | .827 |
| 19.0 | .780 | .842 | .853 |
| 20.0 | .813 | .868 | .880 |
| 21.0 | .845 | .893 | .905 |
| 22.0 | .875 | .917 | .931 |
| 23.0 | .907 | .940 | .957 |
| 24.0 | .932 | .960 | .980 |
| 25.0 | .957 | .975 | .993 |
| 26.0 | .973 | .990 | 0.998 |
| 27.0 | .987 | .995 | 1.000 |
| 28.0 | .995 | 0.998 | .000 |
| 29.0 | 0.997 | 1.000 | .000 |
| 30.0 | 1.000 | .000 | .000 |
| 31.0 | .000 | .000 | .000 |
| 32.0 | 1.000 | 1.000 | 1.000 |

ensions derived on the basis of geometrical elements obtained employing Russell and Merrill's method, and which were published earlier.

The spectrum-luminosity classification of the components has been adopted from our earlier paper (Srivastava, 1981). The absolute visual and the bolometric magnitudes of the primary and the secondary components have been revised on the basis of present geometrical elements obtained employing Kopal's method, using $T(\odot) = 5730$°K, $T_1 = 6150$°K (F8V) and $T_2 = 5220$°K (G9V) (*vide* Arp's, 1958 tables).

The two sets of elements tabulated in Tables 3 and 4 using the same set of observations show slight variations. This may be due to the method of computation of elements involved. In the Russell and Merrill (1952) method only a few crucial points are considered and more weight is given to the points near the shoulder and near bottom of the minima, while in Kopal's (1975a, b, c, d, e) method, all the points of the eclipse are taken into account.

## 6. Roche constants

The revised Roche constants for the equipotential surfaces of AR Lac have been computed, using the elements obtained from Kopal's method, from the relation given by Kopal (1959a), and are given as under:

$$C_0 = 4.0, \quad C_1 = 7.5, \quad C_2 = 4.3,$$

$$C_1/C_2 = 1.74, \quad C_2/C_0 = 1.10, \quad C_1/C_0 = 1.87.$$

The value of $C_0$ having been read-out from the table given by Kopal (1959b) corresponding to a mass ratio of 1.008 ($\approx 1.01$). The values of Roche constants are similar to those based on the elements obtained employing Russell and Merrill (1952) method. Also, the values of Roche constants are in agreement with those derived by Kopal and Shapley (1956). Since $C_1 > C_0$ and $C_2 > C_0$, we infer that the system is a detached one.

## 7. Roche radii

The Roche radii for the primary and the secondary components of AR Lac have been determined from the relation (Paczyński, 1971):

$$\frac{r_1}{a} = 0.38 + 0.2 \log \frac{1}{q}, \quad \left(q = \frac{m_2}{m_1}\right)$$

$$\frac{r_2}{a} = 0.38 + 0.2 \log q,$$

which is valid for systems satisfying the condition $0.3 < q < 20$. Here $r_1$ and $r_2$ are the radii (in terms of the separation of their centres), $m_1$ and $m_2$ are the masses of the components and $a$ is the separation between them. The values of radii and Roche radii of the components are given as under:

Primary component $r = 0.17$, $r^* = 0.38$,
Secondary component $r = 0.34$, $r^* = 0.38$.
The Roche radii ($r^*$) are schematically shown in Fig. 3.

## 8. Evolutionary discussion

The concerned values are plotted on the $\left[\log\left(\frac{m}{m_\odot}\right), \log\left(\frac{R}{R_\odot}\right)\right]$ relation (Fig. 1) and on the [Spectral type, $M(bol)$] relation (Fig. 2) valid for systems belonging to the Main Sequence (Kopal 1955; Arp, 1958). The solid line in Figs. 1 and 2 represent the average trend of the Main Sequence.

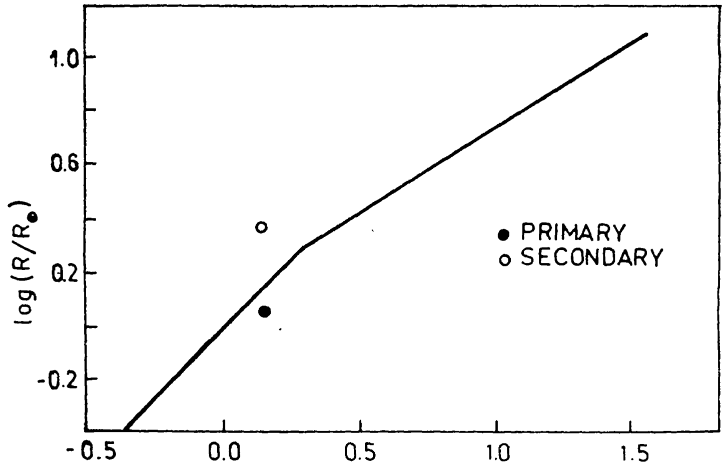

Fig. 1. Position of components of AR Lacertae on the mass-radius relation for close binary systems with detached components. The line on the diagram represents the average trend of the Main Sequence (Kopal, 1955).

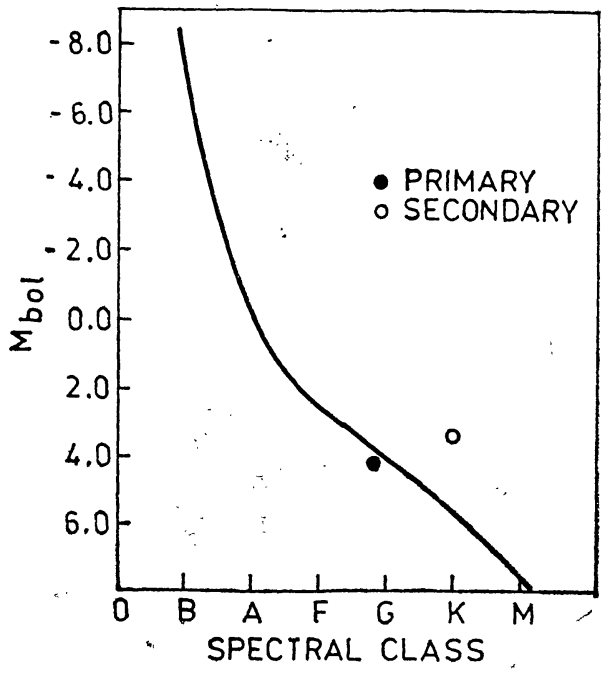

Fig. 2. Position of components of AR Lacertae on spectral class-luminosity relation for the Main Sequence stars. The continuous line on the diagram schematizes the average trend of the Main Sequence (Arp, 1958).

(i). Looking at Figs. 1 and 2, it is evident that the secondary component of AR Lac is away from the Main-Sequence, while the primary component lies nearly on the Main-Sequence.

(ii). The Roche constants show that $C_2/C_0 \approx 1$. This fact suggests that AR Lac may be in a semi-detached state of evolution, and may be possessing a sub-giant secondary (*vide* Kopal, 1959c).

(iii). An examination of Fig. 3 reveals that the primary component of AR Lac is far from filling its Roche lobe. However, $r^* \approx r$, for the secondary component. These facts suggest that either the secondary component of AR Lac is approaching to fill its Roche lobe or else that the secondary component of the system, after filling its Roche lobe, has started shrinking.

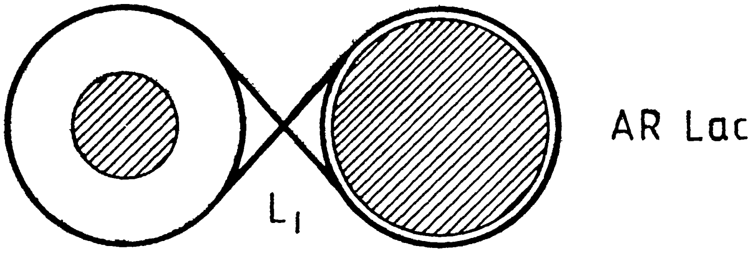

Fig. 3. Roche diagram in terms of separation of the components as unit. The primary component is shown on the left. The Lagrangian point $L_1$ is also shown in the figure.

(iv). There is evidence that the secondary component of AR Lac is losing mass to the primary component, and is thereby producing changes in the period.

(v). The spectral-luminosity classification of the secondary component of AR Lac has been given by us as G9V-IV (vide Srivastava, 1981). This fact is further supported by the position of the secondary component of AR Lac in spectral-luminosity plot wherein the secondary component lies near the sub-giant branch (*vide* Fig. 21, Dufay, 1964).

(vi). The system AR Lac has been placed among the eclipsing system with "undersize" sub-giant secondary (*vide* Kopal, 1959d).

(vii). Theokas (1977) has stated that Schneller (1962) lists AR Lac as being between the detached and semi-detached state of evolution. Kopal (1959c) explains that systems with secondary sub-giants, while being detached, are near or actually in contact with the inside surface of the Roche lobe. To resolve this is admittedly difficult.

In conclusion, we may say that the primary component of AR Lac is a Main-Sequence star and the secondary component is in an advanced stage of evolution. Also, the secondary component of AR Lac is a subgiant. Lastly, although apparently, the system appears to be a detached one, the possibility of it being in a semi-detached state of evolution can not be ruled out in the light of above discussion.

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