AN ANALYSIS OF THE SPECTRUM OF THE ZETA GEMINORUM

B. B. SANWAL, B. S. RAUTELA, and G. C. JOSHI

Uttar Pradesh State Observatory, Manora Peak, Naini Tal, India

(Received 16 December, 1985)

Abstract. Four spectrograms of the cepheid ζ Geminorum at different phases have been analysed for the determination of the abundances of various elements. The analysis shows that the atmosphere of ζ Gem has an essentially solar composition.

1. Introduction

Studies of the chemical composition of stars are very important in understanding the nucleosynthesis processes, occurring in the stellar interiors. Minor differences in the chemical composition of stars at various evolutionary stages and different places of formation can provide information about stellar as well as galactic evolution as a whole. The classical cepheids constitute an important class of objects occupying the instability strip in the H–R diagram representing sufficiently advanced stage of stellar evolution. The amplitudes and the periods of pulsation of these stars increase as their position moves up in the instability strip. In other words the cepheids with longer periods and larger amplitudes have longer evolutionary ages away from the main sequence. This difference in the evolutionary ages is expected to produce differences in the chemical composition amongst these stars.

In the past, abundance analyses of cepheids have generally been carried out through curve of growth method (Rodgers and Bell, 1963; Bappu and Raghavan, 1969; Schmidt, 1971; Van Paradijs, 1971; Rautela *et al.*, 1981; and others). Luck and Lambert (1981) and Giridhar (1983) have analysed a few cepheids using spectrum synthesis techniques. The latter technique is more accepted and is preferred over the curve of growth method, provided spectra of high resolution are available and accurate *gf* values for the lines are available. The resolution of the spectra used in the Luck and Lambert (1981) study was 0.2 Å whereas it was 0.4 Å in the study of Giridhar (1983). Abundances derived by Luck and Lambert (1981) are reported to be slightly higher by Hearnshaw and Desikachary (1982) and by Giridhar (1983).

Luck and Lambert (1981) and Giridhar (1983) have calculated the oscillator strengths for metallic lines from solar equivalent widths taking a solar model atmosphere and depth independent microturbulance. Using the oscillator strengths thus calculated the derived abundances of elements for a star depend on solar parameters also. Here we have attempted the analysis of the spectrum of ζ Gem, using absolute values of oscillator strengths that were available and the model stellar atmospheres given by Kurucz (1979). The Cepheid variable ζ Gem has magnitude variation ranging from 3.66 to 4.16 and

spectral type varies from F7Ib to G3Ib. The period of light cycle variation is 10.15082 days. The epoch used in computing the phases of the star has been taken from Kukarkin *et al.* (1969).

2. Observational Material

The observational material for the present study consists of four spectrograms of the star taken at a dispersion of 4.4 Å mm⁻¹ and covering the wavelength range approximately $\lambda 5000 - \lambda 6800$ Å. These spectrograms were taken by Dr E. G. Schmidt and his co-workers at Kitt Peak National Observatory using the 84" Coudé spectrograph and recorded on the 098–02 emulsion. The spectrograms were kindly loaned to us by Dr Schmidt.

Details of spectrograms are given in Table I.

TABLE I

Details of spectrograms

Plate No.	Date	Exposure U.	Phase	
		Beginning	End	
2502	14 Nov., 1970	9.55	11.05	0.17
2503	14 Nov., 1970	11.25	12.46	0.18
2607	5 Feb., 1971	6.52	7.18	0.34
2612	6 Feb., 1971	6.25	7.22	0.43

Density tracings for these plates have been obtained using Zeiss microphotometer. The continuum level of these tracings were found by drawing lines through points that have been located as line free or almost line free continuum points in the photometric atlas of the spectrum of Arcturus (Griffin, 1968) and compared with our tracings. The line lists given by Hearnshaw and Desikachary (1982) and Luck (1977) and photometric atlas of the spectrum of Arcturus (Griffin, 1968) have been used for the identification and selection of lines for our study. The densities of the lines used by us for this study have been converted into intensities through necessary calibrations provided with the plates. The intensity curves of the lines of interest have been planimetered to get the equivalent widths.

3. Selection of Stellar Model Atmosphere

The basic requirement of the model atmosphere based fine analysis is to choose a model atmosphere that most closely resembles the stellar atmosphere to be investigated. This is usually done by the correct choise of the effective temperature, $\log g$ and chemical composition of the model atmosphere. ζ Gem has been studied photometrically by

Mitchell et al. (1964) and spectrophotometrically by Rautela and Joshi (1983). The effective temperature of the star varies between 5625 and 6200 K with over all maximum error of about \pm 220 K. Rautela and Joshi (1983) have taken an average $\log g = 1.8$ for the star, whereas Luck and Lambert (1981) have derived $\log g = 1.5$ at phases 0.17 and 0.78 and $\log g = 1.9$ at phase 0.58. Based on the above, we have chosen Kurucz (1979) model atmospheres for effective temperature 6000 K and $\log g = 1.5$ for plates representing phases 0.17 and 0.18. For other two spectrograms at phases 0.34 and 0.43 we have taken model having effective temperature 5500 K and $\log g = 1.5$. Models have solar chemical composition.

Although recent models are adopted, they still fall short of perfection in the representation of cepheid atmospheres. The effect of pulsation on the atmospheres, departure from plane-parallelism, non LTE phenomena and the depth and phase variations of the turbulent velocity are neglected. The departure from LTE in low gravity stars has been studied by Lites and Cowly (1974). They have shown that for the lines of moderate strength, the departure from LTE causes only a small difference in the equivalent widths and, hence, the assumption of LTE does not introduce significant errors in abundances if strong lines are excluded.

Luck and Lambert (1981) have taken the microturbulent velocity as 3 and 4 km s⁻¹, respectively, for the corresponding two models whereas Giridhar (1983) has taken it to be 6 and 4.5 km s⁻¹ for the same. We have adopted 4 km s⁻¹ as a mean value of the microturbulence for the phases considered by us.

Experimentally determined accurate gf values for the lines under investigation have been taken from different sources, because there is no such single source which can provide gf values for all the lines. Where the experimentally determined gf values are not available, we have used the values as calculated by Kurucz and Peytremann (1975). Table II gives the gf values used in our analysis and the sources of these values.

4. Method of Analysis

To carry-out an abundance analysis, detailed computations of the equivalent widths of selected lines have been done for different assumed abundances. The computed equivalent widths are then plotted against the assumed abundances. The observed equivalent width of a line of the element under investigation is then entered in this plot and the abundance is read off. The weak line method used by Waddell (1958) has been modified for the case of stars and has been used in the present work. The method is as follows.

The residual intensity at a point on the line profile at a wavelength λ and at a disc position $\mu(=\cos\theta)$ is defined by

$$R_{\mu, \lambda} = \left(1 - \frac{\text{line intensity at wavelength } \lambda}{\text{continuum intensity at line centre}}\right), \tag{1}$$

TABLE II

Wave- length	χ (eV)	log gf	Ref.	Measu	red equi	valent wi	dths	Derive	ed abund	ances	
(nm)					$(-\log W/\lambda)$						
				Phase				Phase			
				0.17	0.18	0.34	0.43	0.17	0.18	0.34	0.43
					Fe	I					
565.23	4.26	-1.85	9	5.20	5.15	5.27	5.05	7.73	7.79	7.28	7.64
570.55	4.30	- 1.98	7	5.20	5.03	5.02	4.99	7.85	7.94	7.83	7.88
575.20	4.55	- 1.14	7	4.97	4.93	4.81	4.77	7.40	7.44	7.26	7.30
591.63	2.45	- 2.90	9	4.82	4.82	4.66	4.61	7.81	7.81	7.87	8.08
592.78	4.65	- 0.99	9	5.10	4.90	4.87	4.86	7.36	7.78	7.83	7.88
593.47	3.93	- 1.07	9	4.76	4.73	4.59	4.59	7.61	7.71	7.98	7.98
595.27	3.98	- 1.34	9	4.81	4.77	4.68	_	7.73	7.85	7.85	_
607.90	4.65	- 1.02	9	5.00	4.97	4.95	4.84	7.60	7.66	7.70	7.90
609.67	3.98	- 1.82	9	5.15	5.05	4.96	4.84	7.53	7.69	7.83	8.00
616.54	4.14	- 1.46	9	5.00	5.02	5.13	4.98	7.57	7.54	7.48	7.32
618.02	2.73	- 2.68	9	4.81	4.71	4.73	4.65	7.88	8.09	7.73	8.00
620.03	2.61	- 2.39	9	4.85	4.76	4.72	4.55	7.35	7.67	7.37	8.02
622.92	2.84	- 2.99	7	4.93	4.91	4.76	4.77	7.97	8.00	7.97	7.94
630.25	3.69	- 1.50	7	4.80	4.75	4.78	4.63	7.70	7.86	7.38	8.00
635.50	2.84	-2.32	9	4.80	4.69	4.69	4.57	7.70 7.49	8.15	7.74	8.16
656.92	4.73	- 2.32 - 0.38	9	4.71	4.68	4.68	4.78	7.49	8.01	7.77	7.32
659.39	2.43	-0.38 -2.43	9	4.69	4.64	4.58	4.78	7.87	8.08	8.03	7.82
659.76	4.79	-0.97	9	5.00	4.94	5.14	4.99	7.41	7.92	7.12	7.42
					Fe	11					
(00.41	2.20	111	7	170			475	7.62	7.00	7.22	751
608.41	3.20	- 4.11 - 4.47	7	4.78 4.90	4.68	5.00	4.75	7.63	7.98	7.33	7.54
611.33	3.22		10		5.00	4.97	4.93	7.69	7.46	7.62	7.64
622.93	2.83	-4.82	10	4.83	4.90	4.78	4.74	7.83	7.67	7.66	7.83
623.94	2.81	- 4.54	10	4.96	4.93	5.11	5.02	7.18	7.04	7.26	7.16
645.64 651.61	3.90 2.89	- 2.26 - 3.18	10 10	4.56 4.58	4.58 4.55	4.58 4.50	4.51 4.58	7.96 7.48	7.89 7.67	7.63 7.72	7.76 7.51
031.01	2.07	5.10	10	4.50			7.50	7.40	7.07	1.12	7.51
					Mg	ĮΙ					
571.11	4.34	- 1.69	5	4.81	4.81	4.66	4.60	7.18	7.18	7.35	7.58
631.92	5.11	- 2.20	8	-	5.27	5.12	5.11	_	7.62	7.62	7.60
					Ca	I					
560.13	2.52	- 0.69	12	4.88	4.80	4.62	4.64	5.65	5.98	6.42	6.31
610.27	1.88	-0.79	11	4.67	4.58	4.59	4.45	6.03	6.58	6.06	7.14
612.22	1.89	-0.41	12	4.52	4.58	4.55	4.38	6.51	6.08	5.81	7.87
616.22	1.90	- 0.09	11	4.56	4.62	4.60	4.47	5.69	5.46	5.89	6.55
616.64	2.52	-1.30	8	4.94	4.86	4.96	4.60	6.35	6.54	5.89	6.75
616.91	2.51	-0.55	12	4.70	4.70	4.72	4.62	6.28	6.28	5.78	6.33
616.96	2.51	-0.27	12	4.68	4.70	4.59	4.51	6.08	6.04	5.56	7.06
643.91	2.53	+0.47	12	4.59	4.69	4.48	4.49	6.50	6.54	6.67	6.97
645.56	2.52	-1.54	8	4.98	4.80	4.82	4.91	6.34	6.84	6.59	6.37

Table II (continued)

Wave- length	χ (eV)	log gf	Ref.	Meası	ıred equi	valent wi	dths	Derive	ed abund	ances	
(nm)	(ev)				(– lo	$g W/\lambda$)					
				Phase				Phase			
				0.17	0.18	0.34	0.43	0.17	0.18	0.34	0.43
					Sc	II					
552.68	1.77	+ 0.06	7	4.53	4.48	4.65	4.49	2.99	3.32	3.28	2.65
555.22	1.45	-2.37	7	5.15	5.24	4.96	5.05	3.22	3.11	3.38	3.24
566.72	1.50	-1.24	7	4.74	4.73	4.74	4.65	2.95	2.97	2.81	3.22
624.57	1.51	- 0.98	13	4.59	4.64	4.62	4.53	3.48	3.23	3.22	3.76
660.46	1.35	- 1.48	13	4.69	4.67	4.59	4.74	3.33	3.41	3.74	3.14
					Ti	I					
568.95	2.30	- 0.29	7	5.34	5.27	5.16	5.13	5.29	5.37	5.13	5.17
586.65	1.07	-0.84	7	4.99	4.97	4.73	4.80	5.20	5.23	5.33	5.16
589.93	1.05	- 1.15	3	4.96	4.84	4.74	4.69	5.51	5.67	5.51	5.61
592.21	1.05	- 1.47	3	5.22	5.09	4.84	5.05	5.46	5.61	5.46	5.23
612.62	1.07	- 1.16	7	4.95	5.20	5.03	4.97	5.54	5.20	5.01	5.12
630.38	1.44	- 1.76	7	5.31	5.52	5.02	5.06	5.50	5.32	4.96	4.88
					v	I					
567.09	1.08	- 1.12	7	4.45	5.50	5.26	5.01	4.78	4.73	4.57	4.86
569.85	1.06	-0.37	7	4.87	4.88	4.78	4.79	4.79	4.79	4.59	4.58
570.36	1.05	- 0.38	7	5.07	5.25	4.94	5.01	4.55	4.31	4.28	4.15
572.71	1.08	+ 0.05	7	4.88	4.95	4.76	4.74	4.52	4.38	4.31	4.38
608.14	1.05	- 0.51	2	5.35	5.07	4.89	5.06	4.32	4.68	4.53	4.21
619.92	0.29	- 1.61	7	5.22	5.51	5.35	4.93	4.83	4.51	4.13	4.62
					Cr	I					
578.18	3.32	- 0.80	1	4.90	4.88	4.99	4.99	6.42	6.45	6.06	6.08
578.80	3.32	- 0.14	1	5.00	5.26	4.87	4.82	5.74	5.25	5.69	5.84
					Si	I					
56156	4.02	2.14	(107	5.00	4.00	4.07	7.01	7.4	7.71	7.71
564.56	4.93	- 2.14	6	4.87	5.09	4.89	4.86	7.91	7.64	7.71	7.76
569.04	4.93	- 1.87	6	4.91	4.99	5.01	4.88	7.65	7.53	7.21	7.46
577.21	5.08	- 1.75	6	4.93	4.96	4.86	4.83	7.66	7.61	7.57	7.63
579.31	4.93	- 1.48	12	4.89	4.96	4.85	4.80	7.35	7.18	7.11	7.26
594.86	5.06	- 1.23	6	4.70	4.80	4.81	-	7.74	7.46	7.11	-
					Co	I					
555.38	1.71	- 2.06	4	5.70	5.70	5.06	5.08	4.86	4.86	5.12	5.08
618.90	1.71	-2.45	4	5.58	5.50	5.41	5.24	5.36	5.46	5.05	5.26
628.26	1.74	-2.16	7	5.43	5.38	5.31	5.07	5.29	5.36	5.94	5.08

Table II (continued)

Wave-	χ	$\log gf$	Ref.	Measu	ired equi	valent wi	dths	Derive	ed abund	ances	
length (nm)	(eV)				(– lo	g <i>W</i> /λ)					
				Phase				Phase			
				0.17	0.18	0.34	0.43	0.17	0.18	0.34	0.43
					Ni	I					
559.37	3.90	- 0.84	7	5.02	4.92	4.99	4.86	6.30	6.45	6.30	6.13
560.00	4.09	-1.08	7	4.82	5.05	4.96	4.93	6.74	6.97	6.47	6.51
574.84	1.68	-3.54	7	_	_	4.93	4.91	_		6.51	6.54
611.11	4.09	-1.18	7	5.29	_	5.10	4.97	6.41	_	6.18	6.38
617.54	4.09	-0.68	7	4.93	4.91	4.97	4.84	6.48	6.51	6.11	6.41
653.29	1.93	- 3.59	7	-	-	5.28	5.37	-	-	6.37	6.27

- 1. Biemont (1977)
- 2. Biemont (1978)
- 3. Blackwell et al. (1982)
- 4. Cardon et al. (1982)
- 5. Froese-Fischer 1975)
- 6. Garz (1973)
- 7. Kurucz and Peytremann (1975)
- 8. Lambert and Luck (1978)
- 9. May et al. (1974)
- 10. Phillips (1979)
- 11. Smith and O'Neill (1975)
- 12. Weise et al. (1969)
- 13. Weise et al. (1975)

which can be written (cf. Waddell, 1958) as

$$R_{\mu, \lambda} = \int_{0}^{\infty} \psi(\mu) \left(\frac{K_{\lambda}}{K_{\lambda_{0}}}\right) g(\mu) d\left(\frac{\tau}{\mu}\right), \tag{2}$$

where $g(\mu)$, weighting factor; K_{λ} , line opacity at wavelength λ on the line profile; K_{λ_0} , continuum opacity; τ , optical depth.

Using the expression for the residual intensity (Equation (2)) the equivalent width is written as

$$W_{\lambda, \mu} = \int_{-\infty}^{\infty} \int_{0}^{\infty} \psi(\mu) g(\mu) \left(\frac{K_{\lambda}}{K_{\lambda_0}}\right) d\left(\frac{\tau_{\lambda}}{\mu}\right) d\lambda, \qquad (3)$$

where the saturation factor $\psi(\mu)$ is given by

$$\psi(\mu) = \exp\left[-\int_{0}^{\tau} \frac{K_{\lambda}}{K_{\lambda_{0}}} d\left(\frac{\tau_{\lambda}}{\mu}\right)\right]. \tag{4}$$

Waddell (1958) suggests that the effect of saturation in weak lines should also be considered. Though the equivalent widths of most of the lines in the present study fall on the linear part of the curve of growth, still the effect of saturation has been taken into account.

The weighting function $g(\tau, \mu)$ is given by

$$g(\tau, \mu) = \frac{\int_{\tau}^{\infty} B(\tau) e^{-\tau/\mu} d\left(\frac{\tau}{\mu}\right) - B(\tau) e^{-\tau/\mu}}{\int_{0}^{\infty} B(\tau) e^{-\tau/\mu} d\left(\frac{\tau}{\mu}\right)},$$
(5)

where $B(\tau)$ is the source function which is taken as Planckian, assuming the case of a gas in thermodynamic equilibrium.

The selective absorption coefficient K_{λ} per atom of hydrogen may be explicitly written as

$$K_{\lambda} = \frac{\sqrt{\pi} e^2 \lambda_0^2}{mc^2} A f \frac{N_{ia}}{N_a \Delta \lambda_D} \exp - \left(\frac{\Delta \lambda}{\Delta \lambda_D}\right)^2, \tag{6}$$

where A is the number of atoms of element A per hydrogen atom; f, the oscillator strength of transition in question; N_{ia} , relative number of atoms of element A at the lower energy level of the transition in question; N_a , the total number of atoms of element A and $\Delta \lambda_D$ is the Doppler broadening given by

$$\Delta \lambda_{\rm D} = \frac{\lambda_0}{c} \sqrt{\frac{2kT}{m_s}} + \xi^2 \,, \tag{7}$$

where λ_0 , central wavelength; c, velocity of light; k, Boltzmann constant; m_s , atomic mass of the species; and ξ , microturbulence velocity.

Waddell (1958) has used the above formulation for a solar problem. For the case of stars we need the radiation integrated over the entire disk of the star. Thus the equivalent width is obtained by integrating over the line and then over the entire disk of the star. Using Aller (1960) we can do so from

$$\frac{W_{\lambda}}{\lambda} = \frac{1}{\lambda} \int_{-\infty}^{\infty} \frac{F_{\text{cont.}} - F_{\nu}}{F_{\text{cont.}}} d(\Delta \lambda).$$
 (8)

Now, since

$$\pi F_{\nu} = 2\pi \int_{0}^{1} I_{\text{line}}(0, \mu) \mu \, d\mu \,, \tag{9}$$

$$\pi F_{\lambda} = 2\pi \int_{0}^{1} I_{\text{cont.}}(0, \mu) \mu \, d\mu$$
 (10)

and

$$W_{\lambda, \mu} = \int_{-\infty}^{\infty} \frac{I_{\mu, \text{ cont.}} - I_{\mu, \lambda}}{I_{\mu, \text{ cont.}}} d(\Delta \lambda), \qquad (11)$$

from Equation (8) we get

$$\frac{W_{\lambda}}{\lambda} = \int_{0}^{1} \frac{W_{\lambda, \mu}}{\lambda} \frac{I_{\mu, \text{ cont.}}}{F_{\text{cont.}}/2} \mu \, d\mu , \qquad (12)$$

where

$$I_{\mu, \text{ cont.}} = \int_{0}^{\infty} B e^{-\tau/\mu} d(\tau/\mu),$$
 (13)

$$F_{\text{cont.}} = 2 \int_{0}^{\infty} BE_2(\tau) d\tau; \qquad (14)$$

and $E_2(\tau)$ is the second integro-exponential function which is calculated using Kourganoff (1952).

To estimate the continuous opacity K_{λ_0} , we have included the following sources of opacity, which commonly operate in the atmosphere of the stars.

- (1) Opacity due to negative hydrogen ion.
- (2) Opacity due to neutral hydrogen.
- (3) Scattering due to H and H_2 .
- (4) Thomson scattering.

We have used the general formulations given by Tsuji (1966) to carry out continuous opacity calculations. Generally in stellar model atmosphere the optical depth is given for the wavelength 500 nm. To obtain the optical depth at any λ the following relationship has been used:

$$\tau_{\lambda} = \int_{0}^{\tau} \frac{K_{\lambda}}{K_{500 \text{ nm}}} \, \mathrm{d}\tau_{500 \text{ nm}} \,. \tag{15}$$

5. Results and Discussions

List of the lines selected in the present study along with the measured equivalent widths and the corresponding derived metal abundances are given in Table II. Table III lists

	TA	BLE III			
Mean	abundances	relative	to	$N_{\mathbf{H}} =$	12.00

Element	Solar	Luck and Lambert (1981)	Present analysis	
Mg	7.6	_	7.5 ± 0.2	
Si	7.6	7.8	7.5 ± 0.2	
Ca	6.3	6.8	6.4 ± 0.5	
Sc	3.1	3.5	3.2 ± 0.3	
Ti	5.0	5.4	5.3 ± 0.2	
V	4.1	4.8	4.5 ± 0.2	
Cr	5.7	6.1	5.9 ± 0.4	
Fe	7.6	7.8	7.7 ± 0.3	
Co	5.0	5.5	5.1 ± 0.2	
Ni	6.3	6.5	6.4 ± 0.2	

the average abundance of various species. Luck and Lambert (1982) and solar (Engvold, 1977) abundance values are also given for comparison. Luck and Lambert (1981) abundance values are slightly higher than our results. The difference may be because of two possible reasons. (1) Measured equivalent width may be on the higher side and (2) the model atmosphere chosen are different. Hearnshaw and Desikachary (1982) have pointed out that the Luck and Lambert equivalent widths are on the higher side, which is reflected on the higher values of the derived abundances. Our analysis shows that the atmosphere of ζ Gem has essentially solar composition. It can again be mentioned that our analysis is independent of solar model atmosphere which has been used to derive gf values by other authors. The chemical composition of the atmosphere of ζ Gem does not exceed the predictions of the standard stellar-evolution calculations, which is in contradiction to the Luck and Lambert (1981) results, but supports the conclusions drawn by Iben and Renzini (1983) that CNO processed elements in cepheids and supergiants do not come up to the surface layers of their atmospheres.

Acknowledgements

We are thankful to Dr S. C. Joshi for guidance and to Dr M. C. Pande for helpful discussions and encouragement. The authors are also thankful to Dr R. E. Luck for providing us his unpublished data on supergiants.

References

Aller, L. H.: 1960, in J. L. Greenstein (ed.), Stellar Atmospheres, University of Chicago Press, Chicago, p. 156. Bappu, M. K. V. and Raghavan, N.: 1969, Monthly Notices Roy. Astron. Soc. 142, 295.

Biemont, E.: 1977, Ph.D. Thesis, Univ. Liège.

Biemont, E.: 1978, Solar Phys. 56, 79.

Blackwell, D. E., Menon, S. L. R., Petford, A. D., and Shallis, M. J.: 1982, Monthly Notices Roy. Astron. Soc. 201, 611.

Cardon, B. L., Smith, P. L., Scalo, J. M., Testerman, L., and Whaling, W.: 1982, Astrophys. J. 260, 395.

Engvold, O.: 1977, Phys. Scripta 16, 48.

Froese-Fischer, C.: 1975, Can. J. Phys. 53, 184.

Garz, T.: 1973, Astron. Astrophys. 26, 471.

Giridhar, S.: 1983, J. Astrophys. Astron. 4, 75.

Griffin, R. F.: 1968, A Photometric Atlas of the Spectrum of Arcturus 3600-8825 Å, Cambridge Philosophical Society, Cambridge.

Hearnshaw, J. B. and Desikachary, K.: 1982, Monthly Notices Roy. Astron. Soc. 198, 316.

Iben, I., Jr. and Renzini, A.: 1983, Ann. Rev. Astron. Astrophys. 21, 271.

Kourganoff, V.: 1952, Basic Methods in Transfer Problems, Oxford University Press, London, p. 254.

Kukarkin, B. V., Kholopov, P. N., Efremov, Yu. N., Kukarkin, N. P., Kurochkin, N. E., Medvedeva, G. I., Perova, N. B., Fedorovich, V. P., and Frolov, M. S.: 1969, *General Catalogue of Variable Stars*, 3rd ed., Nauka, Moscow.

Kurucz, R. L.: 1979, Astrophys. J. Suppl. 40, 1.

Kurucz, R. L. and Peytremann, E.: 1975, Smithsonian Astrophys. Obs. Sp. Rep. 362.

Lambert, D. L. and Luck, R. E.: 1978, Monthly Notices Roy. Astron. Soc. 183, 79.

Lites, B. W. and Cowley, C. R.: 1974, Astron. Astrophys. 31, 36.

Luck, R. E.: 1977, Astrophys. J. 212, 743.

Luck, R. E. and Lambert, D. L.: 1981, Astrophys. J. 245, 1018.

May, M., Richter, J., and Wichelman, J.: 1974, Astron. Astrophys. Suppl. 18, 405.

Mitchell, R. I., Iriarte, B., Steinmetz, D., and Johnson, H. L.: 1964, Bull. Obs. Tonantzintla Tacubaya 3, 153.

Phillips, M. M.: 1979, Astrophys. J. Suppl. 39, 377.

Rautela, B. S., Sanwal, B. B., and Joshi, S. C.: 1981, Astrophys. Space Sci. 77, 369.

Rautela, B. S. and Joshi, S. C.: 1983, J. Astrophys. Astron. 4, 1.

Rodgers, A. W. and Bell, R. A.: 1963, Monthly Notices Roy. Astron. Soc. 125, 487.

Schmidt, E. G.: 1971, Astrophys. J. 170, 109.

Smith, G. and O'Neill, J. A.: 1975, Astron. Astrophys. 38, 1.

Tsuji, T.: 1966, Publ. Astron. Soc. Japan 18, 134.

Van Paradijs, J. A.: 1971, Astron. Astrophys. 11, 229.

Weise, W. L., Smith, M. W., and Glennon, B. M.: 1969, *Atomic Transition Probabilities*, Natn. Std. Ref. Data Series-Natn. Bur. Std., No. 4.

Weise, W. L. and Fuhr, J. R.: 1975, J. Phys. Chem. Ref. Data 4, 263.

Waddell, J. H.: 1958, Astrophys. J. 127, 284.