THE NATURE OF VELOCITY FIELDS IN QUIESCENT PROMINENCES

(Letter to the Editor)

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Abstract. It is shown that for certain definite conditions of symmetry imposed on the permitting magnetic field geometry for an isothermal case in Kippenhahn and Schlüter's (1957) model of a quiescent prominence, any irrotational velocity field would quickly get converted to rotational.

1. Introduction

So far we have investigated the twin problem of irrotational and rotational velocity fields in quiescent prominences considering Kippenhahn and Schlüter's (1957) prominence model as a typical test case. We showed that in both cases, for the respective velocity fields assumed, the magnetic field geometry remains unchanged (Pande and Bondal, 1991a, b). In the case of rotational velocity field geometry, however, a physically possible solution could be obtained for the horizontal plane, i.e., for the $x - y$ plane, but not for the vertical or the $x - z$ plane. This single possibility enabled us to obtain equal velocity contours (Pande and Bondal, 1991b). In addition to this, we could derive the mass density variation along the y-axis. In what follows here, we have adopted the treatment given earlier in a monograph by Baum *et al.* (1958) where they have quoted a paper by Kaplan (1954) and applied it to Kippenhahn and Schlüter's (1957) model of a quiescent prominence to derive a condition, wherein one type of motion, i.e., irrotational gets transformed to rotational.

2. The Equations

The assumptions made here are the same as those in Pande and Bondal (1991a). If we follow Baum *et al.* (1958) where they have quoted a paper by Kaplan (1954) who generalized the known theorem of Thomson in hydrodynamics for the conservation of circulation in magneto-hydrodynamics, it has been shown that in an inhomogeneous magnetic field the circulation should increase with time. As is well known, the circulation (of velocity) is determined by the integral

$$\Gamma = \oint \mathbf{v}\, d\mathbf{r} = \int \operatorname{curl} \mathbf{v}\, d\sigma , \tag{1}$$

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along a closed contour moving together with the medium, σ is the surface embraced by this contour. The change in circulation with time is equal to

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\oint \mathbf{v}\,\mathrm{d}\mathbf{r} = \oint \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\,\mathrm{d}\mathbf{r} + \oint \mathbf{v}\,\frac{\mathrm{d}}{\mathrm{d}t}\,\mathrm{d}\mathbf{r} =$$

$$= \oint \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\,\mathrm{d}\mathbf{r} + \int \mathrm{d}\left(\frac{v^2}{2}\right) = \int_\sigma \mathrm{curl}\,\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\,\mathrm{d}\sigma = 0\,. \tag{2}$$

Now, the equation of motion is

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{4\pi\rho}\,(\mathbf{B}\times \mathrm{curl}\,\mathbf{B}) + \frac{1}{\rho}\,\mathrm{grad}\,\mathbf{p} + \mathbf{g} = 0\,. \tag{3}$$

If we substitute the value of $\mathrm{d}\mathbf{v}/\mathrm{d}t$ from Equation (3) in Equation (2) and remember the relation

$$\mathrm{curl}(\varphi\mathbf{a}) = \varphi\,\mathrm{curl}\,\mathbf{a} + (\mathrm{grad}\,\varphi)\times\mathbf{a}\,, \tag{4}$$

we get, after a number of simple transformations involving the usage of Equation (4), the equation

$$\left[\mathrm{grad}\,\rho\times\mathrm{grad}\left(\mathrm{p} + \frac{\mathrm{B}^2}{8\pi}\right)\right] + \frac{\rho^2}{4\pi}\,\mathrm{curl}\left[\frac{1}{\rho}\,(\mathbf{B}\cdot\nabla)\mathbf{B}\right] = 0\,. \tag{5}$$

Equation (5) is a general equation. We consider a two-dimensional case, in the $x - z$ plane for Kippenhahn and Schlüter's model and get the following equations, remembering that $\partial/\partial y = 0$: the first term on the left-hand side of Equation (5) reduces to

$$\frac{\partial B_z}{\partial x}\,\frac{\partial\rho}{\partial z}\,\frac{B_z}{4\pi}\,,$$

whereas the second term becomes

$$-\frac{\rho}{4\pi}\,B_x\,\frac{\partial^2 B_z}{\partial x^2} + \frac{1}{4\pi}\,\frac{\partial\rho}{\partial x}\,B_x\,\frac{\partial B_z}{\partial x}\,,$$

so that

$$\frac{B_z}{\rho}\,\frac{\partial\rho}{\partial z}\,\frac{\partial B_z}{\partial x} - B_x\,\frac{\partial^2 B_z}{\partial x^2} + \frac{B_x}{\rho}\,\frac{\partial\rho}{\partial x}\,\frac{\partial B_z}{\partial x} = 0\,. \tag{6}$$

We may call Equation (6) an indicial equation for the two-dimensional Kippenhahn and Schlüter's model which will decide the nature of motions. To ascertain this, we substitute the values of $\partial\rho/\partial z$, $\partial B_z/\partial x$, $\partial^2 B_z/\partial x^2$, and $\partial\rho/\partial x$ as given by Kippenhahn and Schlüter's

(1957) solution

$$B_z = B_z(\infty) \tanh\left[\frac{B_z(\infty)}{B_x}\,\frac{x}{2H_0}\right], \tag{7}$$

where H_0 is the isothermal scale height given by RT/mg, R being the gas constant; T, the temperature; m, the molecular weight; and g, the gravitational acceleration on the Sun's surface. $\partial\rho/\partial z$ is equal to $-\rho/H_0$ and the variation of density ρ along x is

$$\rho = \frac{B_z^2(\infty)}{2K}\ \mathrm{sech}^2\left[\frac{B_z(\infty)}{B_x}\,\frac{x}{2H_0}\right], \tag{8}$$

where $K = m/4\pi RT$.

These solutions for B_z and ρ do not satisfy Equation (6) for the model conditions. Since, on substitution we obtain $-mg/2RTH_0B_x$ or $-\frac{1}{2}H_0^2B_x = 0$, which is not true, as neither H_0 nor B_x can be infinite. Hence, Equation (6) is not satisfied.

From the above it follows that the conservation of potential flows in time, i.e., curl $\mathbf{v} = 0$ does not take place. Baum *et al.* (1958) have concluded that in such a case the irrotational motion would quickly get transformed into a rotational one.

3. Results and Discussion

In earlier investigations by us (Pande and Bondal, 1991a, b), we showed that both the irrotational and rotational motions do not have any effect on the magnetic field geometry of Kippenhahn and Schlüter's model of quiescent prominence. Here we have further shown, that for the same model, under definite conditions of symmetry of magnetic field geometry how any initially present irrotational (or potential) velocity field can quickly get transformed to a rotational one. We conclude, therefore, that the existence of rotational velocity field in Kippenhahn and Schlüter's model in the $x - y$ plane as obtained by us (Pande and Bondal, 1991b) is justified by the generalized Thomson's circulation theorem also.

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