IRROTATIONAL MOTION IN QUIESCENT PROMINENCES

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Abstract. On the basis of Kippenhahn and Schlüter's magnetohydrostatic model of a quiescent prominence, an attempt has been made to study the effect of irrotational motion existing in the prominences on the magnetic field pattern in it, introducing an irrotational velocity field. It is found that, under such a condition, the magnetic field geometry in the model does not change.

1. Introduction

Kippenhahn and Schlüter (1957) proposed a magnetohydrostatic model of a quiescent prominence, in which the prominence material is supported in the sagged portion of the magnetic field lines over a bipolar region of sunspots. The condition stipulated in this case was the implicit absence of motions within the prominence. The prominence as a whole remains in a steady state. However, mass motions can be observed in most quiescent prominences indicating that even though the overall shape of the filaments remains constant with time, the material at any point in the prominence may be in motion (Tandberg–Hanssen, 1967). If such motions are to be investigated then one has to stipulate that the support to the prominence material is provided by a model analogous to the Kippenhahn and Schlüter model. A further condition is, that the model adjusts itself in such a manner that the magnetic field does not change in spite of internal motions. These internal motions should then exert an additional pressure reducing the sag or increasing it in the model. This should further modify the magnetic field gradient because of the strong coupling existing between the field and matter on account of the high electrical conductivity present. The assumption of high electrical conductivity conditions has been justified by Nakagawa (1970). By combining Maxwell's equations and Ohm's law for conducting fluid flow, and assuming a constant electrical conductivity σ the temporal behaviour of the magnetic field **B** can be represented by

$$\frac{\partial \mathbf{B}}{\partial t} = \operatorname{curl}(\mathbf{v} \times \mathbf{B}) + (4\pi\sigma)^{-1}\nabla^2\mathbf{B},$$

where **v** is the material velocity of the medium. In the presence of a magnetic field the electrical conductivity becomes a tensor. The components of parallel and perpendicular electrical conductivity for the coronal and prominence material can be approximately given by

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$$\sigma_{\parallel} = 10^{-14} T^{3/2}$$

and

$$\sigma_{\perp} = 10^{-14} T^{3/2} (1 + \omega_B^2 \tau^2{}_c)^{-1},$$

where T is the temperature, ω_B is the angular cyclotron frequency and τ_c the mean time between collisions (Nakagawa, 1970). Studies have however shown (Maltby, 1978) that for prominences $\omega_B \tau_c \gg 1$ and the electrical conductivity is constant for an isothermal medium, so that the component of the conductivity perpendicular to the magnetic field can be neglected.

In the present paper we have tried to investigate the effects of irrotational motion on the magnetic field pattern and have come to the conclusion that such a motion does not affect the magnetic field pattern and so has a direct bearing on the lifetime of the prominence. That is, rotation could lead to eruption of the prominence (Liggett and Zirin, 1984). The effects of rotational motion will be taken up in a later paper.

2. The Equations

Following Kippenhahn and Schlüter (1957) we assume the prominence to be a thin sheet extending along the 'z' axis perpendicular to the solar surface and 'x' perpendicular to the 'z' direction. The sheet is in the y–z plane. The typical dimensions of a quiescent prominence are, length 200,000 km, height 40,000 km but thickness only 5,000 to 10,000 km (Tandberg–Hanssen, 1967).

We assume the prominence medium to be inviscid, incompressible and possessing infinite electrical conductivity. We now stipulate the condition that the motions present, i.e. the velocity field in the prominence is, irrotational. That is, curl $\mathbf{v}$ = 0.

The equation of motion is

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\,\mathbf{grad}\, p + \rho \mathbf{g}_{\odot} + \frac{(\mathrm{curl}\,\mathbf{B}) \times \mathbf{B}}{4\pi\mu}, \tag{1}$$

where ρ is the mass density, $\mathbf{v}$ the material velocity, p the pressure, $\mathbf{g}_{\odot}$ the gravitational acceleration on the Sun, $\mathbf{B}$ the magnetic field vector, μ the magnetic permeability (taken as unity) and

$$\frac{\mathrm{d}}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + (\mathbf{v}.\nabla)$$

Furthermore,

$$\mathrm{div}\,\mathbf{B} = 0, \tag{2}$$

and, for incompressible fluid conditions

$$\operatorname{div} \mathbf{v} = 0. \tag{3}$$

Now, for steady state, $\partial/\partial t = 0$, and so Equation (1) reduces to

$$(\mathbf{v}.\nabla)\mathbf{v} = -\frac{\mathbf{grad}p}{\rho} + \mathbf{g}_\odot + \frac{(\operatorname{curl}\mathbf{B})\times\mathbf{B}}{4\pi\rho}. \tag{4}$$

Using the vector identity

$$\operatorname{grad}(\tfrac{1}{2}v^2) = (\mathbf{v}\times\operatorname{curl}\mathbf{v}) + (\mathbf{v}.\nabla)\mathbf{v}$$

and remembering that $\operatorname{curl}\mathbf{v} = 0$, we get

$$\rho\operatorname{grad}(\tfrac{1}{2}v^2) = -\,\mathbf{grad}\,p + \rho\mathbf{g}_\odot + \frac{(\operatorname{curl}\mathbf{B})\times\mathbf{B}}{4\pi}. \tag{5}$$

Following Kippenhahn and Schlüter (1957), for a two-dimensional case, we assume that only 'x' and 'z' components of the physical parameters exist and also that $\partial/\partial y \equiv 0$, the condition $\operatorname{curl}\mathbf{v} = 0$ gives

$$\frac{\partial v_x}{\partial z} = \frac{\partial v_z}{\partial x}. \tag{6}$$

A further assumption is that $B_x \neq 0$ but a constant, and $B_z \neq 0$. In a thin filament, B_x changes only slightly with 'x' so that B_x can be taken independent of x in the filament. So B_z becomes a function of x. Also, since $\operatorname{div}\mathbf{B} = 0$, we can take B_z independent of the height 'z'. This justifies B_x to be simply a constant. In a filament, B_z is very strongly dependent on 'x' (at the border crossing the infinitely thin filament B_z has a position of discontinuity), while on the other hand the change in B_x is very small ($\partial B_x/\partial x$ vanishes) in the middle of the filament on the basis of symmetry, (Kippenhahn and Schlüter, 1957).

Taking into account the above assumptions, the equation of motion (5) becomes

$$B_x\frac{\partial^2 B_z}{\partial x^2} + \frac{B_z}{H_0}\frac{\partial B_z}{\partial x} - \rho\mathbf{v}\operatorname{grad}\mathbf{v} = 0, \tag{7}$$

where H_0 is the isothermal scale-height in the prominence, given by $H_0 = RT/mg_\odot$ where R is the gas constant, T is the temperature, 'm' is the mass of the hydrogen atom and '$g_\odot$' the gravitational acceleration on the Sun's surface.

Also,

$$\rho\operatorname{grad}(\tfrac{1}{2}v^2) \equiv \rho\mathbf{v}\operatorname{grad}\mathbf{v}. \tag{7a}$$

Under isothermal conditions and for infinite conductivity the total energy of the system, i.e. magnetic plus kinetic, remains constant, so that we can write

TABLE I

Typical values of B_z at various x values. $B_x = 20$ gauss.

x (cm)	$B_z(\infty) = 5$	$B_z(\infty) = 10$	$B_z(\infty) = 15$	$B_z(\infty) = 20$
1.0 E7*	0.32	1.49	3.33	5.85
1.0 E8	2.84	8.58	14.38	19.77
2.5 E8	4.62	9.96	14.99	19.99
5.0 E8	4.95	9.99	15.00	20.00
7.5 E8	4.99	9.99	15.00	20.00
1.0 E9	5.00	10.00	15.00	20.00

* E7, E8 are powers of ten, e.g., 1.0×10^7, 1.0×10^8 and so on.

$$\frac{B_x^2 + B_z^2}{8\pi} + \frac{1}{2}\rho v^2 = \text{constant}. \tag{8}$$

This gives,

$$\frac{1}{8\pi}\left(2B_x \frac{\partial Bx}{\partial x} + 2B_z \frac{\partial B_z}{\partial x}\right) + \frac{1}{2}\frac{\partial}{\partial x}(\rho v^2) = 0,$$

or

$$\frac{1}{4\pi} B_z \frac{\partial B_z}{\partial x} + \frac{1}{2}\frac{\partial}{\partial x}(\rho v^2) = 0,$$

or

$$\frac{B_z}{4\pi}\frac{\partial B_z}{\partial x} = -\frac{1}{2}\frac{\partial}{\partial x}(\rho v^2). \tag{9}$$

Incorporating (9) in Equation (8) and making use of (7a), we get the solution for B_z as

$$\frac{B_z(\infty) + B_z}{B_z(\infty) - B_z} = \exp\left(\frac{x}{H_0 B_x} B_z(\infty)\right) \tag{10}$$

under the following boundary conditions:

at $x = 0$, $\quad B_z = 0$

at $x = \infty$, $\quad B_z \rightarrow B_z(\infty)$, a constant value.

This is the solution of the equation which includes the condition of the presence of irrotational velocity field in a typical quiescent prominence model, of Kippenhahn and Schlüter (1957). In the model here, we have taken $T = 6000$ K, $B_x =$ 20 gauss, $m = 1$, $g_\odot = 3 \times 10^4$ cm sec^{-2} on the Sun's surface. Taking typical values of $B_z(\infty) = 5$, 10, 15 and 20 gauss, B_z has been calculated for x ranging from 10^7 cm to 10^9 cm.

The results obtained are shown in Table I and plotted in Figure 1.

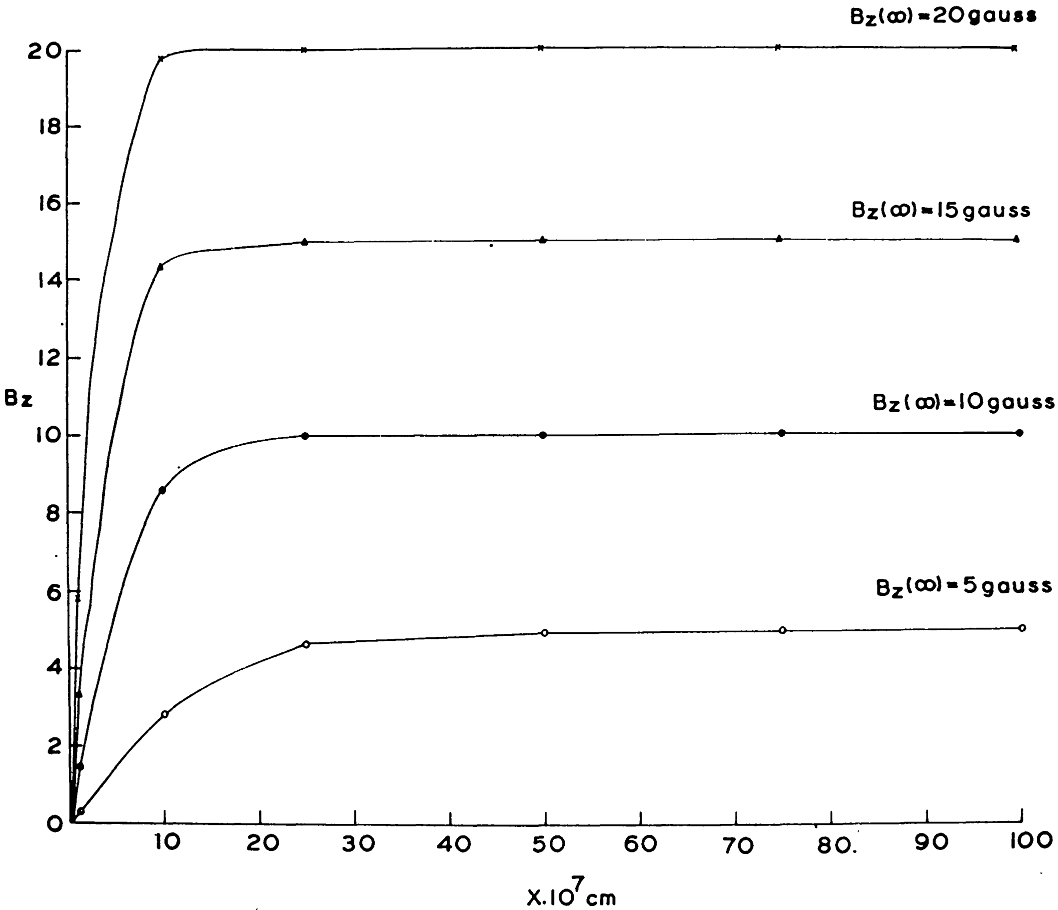

Fig. 1. The supporting magnetic field in Kippenhahn-Schlüter's model of a quiescent prominence for an irrotational velocity field.

3. Results and Discussion

Quiescent prominences are known to have long life times of the order of months. Some are known to survive several solar rotations. The part rendered by the magnetic field is well known, in playing a dominant role in maintaining the stability of quiescent prominences, which exist amidst high temperature surroundings of the solar corona. Observations indicate that though the prominence as a whole remains stationary, the matter in it is continually moving. Whether the motion of matter in the prominence is rotational or irrotational remains to be seen. The nature of this motion perhaps decides the lifetime of the prominence, via its effect on the magnetic field. Rotational motion in many quiescent prominences has been observed in about 10% cases (Liggett and Zirin, 1984). The rotational motion velocity has been reported by these authors to range from 15–75 km sec^{-1}. They however state that this rotation is difficult to understand in view of the concept of the prominence material being suspended in magnetic field. If the field threads the material then the rotation must wind up the magnetic field lines transferring energy to the field and thus slowing down the rotation, unless the lines of force reconnect at the axis of rotation. On the other hand, the restoring forces would try to oppose this twisting. If the restoring forces dominate, the twisting cannot take place, the motion will be irrotational. Twisting of the lines of force will imply rotational motion. The resultant increase in magnetic energy will reach a critical value and will ultimately "break" the lines of force, leading to the disruption of the prominence, and so reducing its lifetime. Since quiescent prominences have a long life time, the velocity field in it can be inferred to be irrotational. So an irrotational velocity field existing in prominences has no effect on the magnetic field pattern.

In our analysis, we introduced an irrotational velocity field and came to the conclusion that this does not affect the magnetic field pattern. This explains, in view of the above reasons, the long lifetime of quiescent prominences. Kippenhahn and Schlüter (1957) solved for the magnetic field geometry without assuming a velocity field. Our analysis, under the condition of irrotational velocity field also reaches the same conclusion. This shows that irrotational velocity field conditions lead to the same conditions as for no velocity field inside the prominence. That is, even if a velocity field is present in quiescent prominences, to account for their long lifetime, the field has to be irrotational in nature.

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