THE ROTATIONAL MOTION IN QUIESCENT PROMINENCES

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Abstract. On the basis of Kippenhahn and Schlüter's magnetohydrostatic model of a quiescent prominence we have attempted to study the effect of a rotational velocity field in it. We find that a physically plausible solution is not possible in the vertical plane. A possibility, however, is shown in the horizontal plane, with certain assumptions to get equal velocity contours.

1. Introduction

We had been investigating the question of velocity fields in quiescent prominences. Starting with Kippenhahn and Schlüter (KS) model (1957) and with the assumption of an irrotational velocity field, we have shown (Pande and Bondal, 1991) that such a field does not change the magnetic field geometry of that model. Retaining the same model, we have here attempted the possibility of accommodating a rotational velocity field in it. We could not find a physically possible solution for a coplanar rotational velocity field in the x–z plane; i.e., in the vertical plane. However, it is found that a rotational velocity field may possibly be accommodated in the model in the x–y plane – i.e. in the horizontal plane – with certain assumptions made to get equal contours.

2. Formulation of the Problem

The assumptions made by us are the same as those in Pande and Bondal (1991) excepting for the velocity field, which we have here assumed to be rotational. The equations are

$$\rho \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\overrightarrow{\mathrm{grad}}\,\vec{p} + \rho\vec{g} + \frac{(\mathrm{curl}\,\vec{B}) \times \vec{B}}{4\pi\mu}, \tag{1}$$

where ρ is the mass density, $\vec{v}$ the mass velocity, p the pressure, $\vec{g}$ the gravitational acceleration on the Sun, $\vec{B}$ the magnetic field vector, μ the magnetic permeability, taken here as unity, and

$$\frac{\mathrm{d}}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + (\vec{v} \cdot \nabla). \tag{2}$$

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Furthermore,

$$\text{div}\,\vec{B} = 0\,; \tag{3}$$

and, for an incompressible fluid,

$$\text{div}\,\vec{\mathbf{V}} = 0\,. \tag{4}$$

Now, for steady-state conditions,

$$4\pi\rho(\vec{\mathbf{V}}\cdot\nabla)\vec{\mathbf{V}} = -4\pi\overrightarrow{\text{grad}}\,p + 4\pi\vec{g}\rho + (\text{curl}\,\vec{B})\times\vec{\mathbf{B}}\,. \tag{5}$$

The velocity field, now assumed to be rotational is taken to be of the form (Smirnov, 1962)

$$v_x = \alpha_{11}x - a_{12}z$$

and (6)

$$v_z = a_{12}x - \alpha_{11}z\,,$$

where a_{11} and a_{12} are the coefficients to be determined and also

$$v_x = a_{11}x - a_{12}y$$

and (6a)

$$v_y = a_{12}x - \alpha_{11}y\,.$$

It has been shown by Smirnov (1962) that, under certain conditions and assumptions of the velocity field made, leads to elliptically-closed equal velocity lines with the origin being a singular point.

To derive elliptically-closed equal velocity contours, the method given by Yefimov (1954) has been used for obtaining the necessary iso-velocity ellipses.

3. Possibility of Rotational Motion in the *x–z* Plane

Equation (1) is written as

$$4\pi\rho(\vec{\mathbf{V}}\cdot\nabla)\vec{\mathbf{V}} = -4\pi\,\overrightarrow{\text{grad}}\,p + 4\pi\rho\vec{g} + (\text{curl}\,\vec{B})\times\vec{\mathbf{B}}\,.$$

Using Eqs. (6) and (6a) above, and putting

$$S \equiv 4\pi(a_{11}^2 - a_{12}^2), \qquad N \equiv \frac{4\pi RT}{m},$$

where R is the gas constant, T the temperature and m the mass of the hydrogen atom, we obtain

$$Sz\rho = -N\frac{\partial \rho}{\partial z} + B_z\frac{\partial B_z}{\partial x} + 4\pi\rho g \tag{7}$$

and

$$Sx\rho = -N\frac{\partial \rho}{\partial x} - B_z\frac{\partial B_z}{\partial x}. \tag{8}$$

Differentiating Equation (7) w.r.t. x and Equation (8) w.r.t. z, and subtracting, we finally get

$$B_x\frac{\partial^2 B_z}{\partial x^2} + \frac{(Sz - 4\pi g)}{N}B_z\frac{\partial B_z}{\partial x} =$$

$$= S_x\frac{\partial \rho}{\partial z} - (Sz - 4\pi g)\frac{Sx\rho}{N}. \tag{9}$$

Though we could obtain iso-velocity contours in the x–z plane, but the solution for B_z as a function of z, would contain a dependence of B_z on z, which violates the initial assumption inherent in the KS model and also the condition div $\vec{B} = 0$. This shows that a rotational velocity field in the x–z plane in the KS model is not physically possible or should be accommodated in some alternate way is not known to us yet. The details of this case are therefore being omitted.

4. Possibility of Rotational Motion in the x–y Plane

After having ascertained that the rotation iso-velocity contours cannot be accommodated in the KS model in the x–z plane, the next problem taken up is to find out whether the same can be reconciled in the x–y plane. Using Equations (1)–(5) and (6a), we arrive at the following three equations for the x, y and z components of Equation (1):

$$S\rho x = +\frac{4\pi RT}{m}\frac{\partial \rho}{\partial x} - B_z\frac{\partial B_z}{\partial x}, \tag{10}$$

$$S\rho y = -\frac{4\pi RT}{m}\frac{\partial \rho}{\partial y} \tag{11}$$

and

$$0 = -\frac{4\pi RT}{m}\frac{\partial \rho}{\partial z} + 4\pi\rho g + B_z\frac{\partial B_z}{\partial x}. \tag{12}$$

Differentiating Equation (10) w.r.t. z and Equation (12) w.r.t. x and subtracting, we arrive at the following equations, after some rearrangement:

$$B_x \frac{\partial^2 B_z}{\partial x^2} + \frac{4\pi g}{N} B_z \frac{\partial B_z}{\partial x} + \frac{4\pi g S \rho x}{N} + S \frac{\partial \rho}{\partial z} x = 0 . \tag{13}$$

Then, if we consider the last two terms on the L.H.S. of Equation (13), we obtain the term

$$Sx\rho \left(\frac{mg}{RT} + \frac{1}{\rho} \frac{\partial \rho}{\partial z} \right)$$

and find that the terms in the brackets vanish, since

$$\frac{1}{\rho} \frac{\partial \rho}{\partial z} = - \frac{mg}{RT} ,$$

and the final equation for the magnetic field remains the same as in the KS model. Hence, B_z can be represented as

$$B_z = B_z(\infty) \tanh \left(\frac{B_z(\infty)}{B_x} \frac{x}{2h} \right) ,$$

where

$$\frac{1}{hB_x} = \frac{mg}{RTB_x} .$$

In brief, the introduction of a rotational velocity field in the x–y plane does not affect the magnetic field geometry of the KS model. Also, the variation of density with x remains the same (Kippenhahn and Schlüter, 1957). In addition to this, the rotational velocity field assumed in the x–y plane would impress a density variation along the y-axis as given by

$$\rho = \rho_0 \exp \left(- \frac{mS}{4\pi RT} \right) y \tag{14}$$

which follows from Equation (11).

The iso-velocity contours in the x–y plane were calculated after appropriately choosing the coefficients a_{11} and a_{12} at discrete values of velocities ranging between 10–50 km sec^{-1}.

5. Results and Discussion

It has been found that elliptically closed iso-velocity contours for KS model in x–z plane could not be obtained by us, since the condition div $\vec{B} = 0$ was being violated, with the assumed velocity fields and the density distribution. However, if iso-velocity contours are traced for the x–y plane, the magnetic field geometry of the KS model is not affected though the solutions for this case are possible.

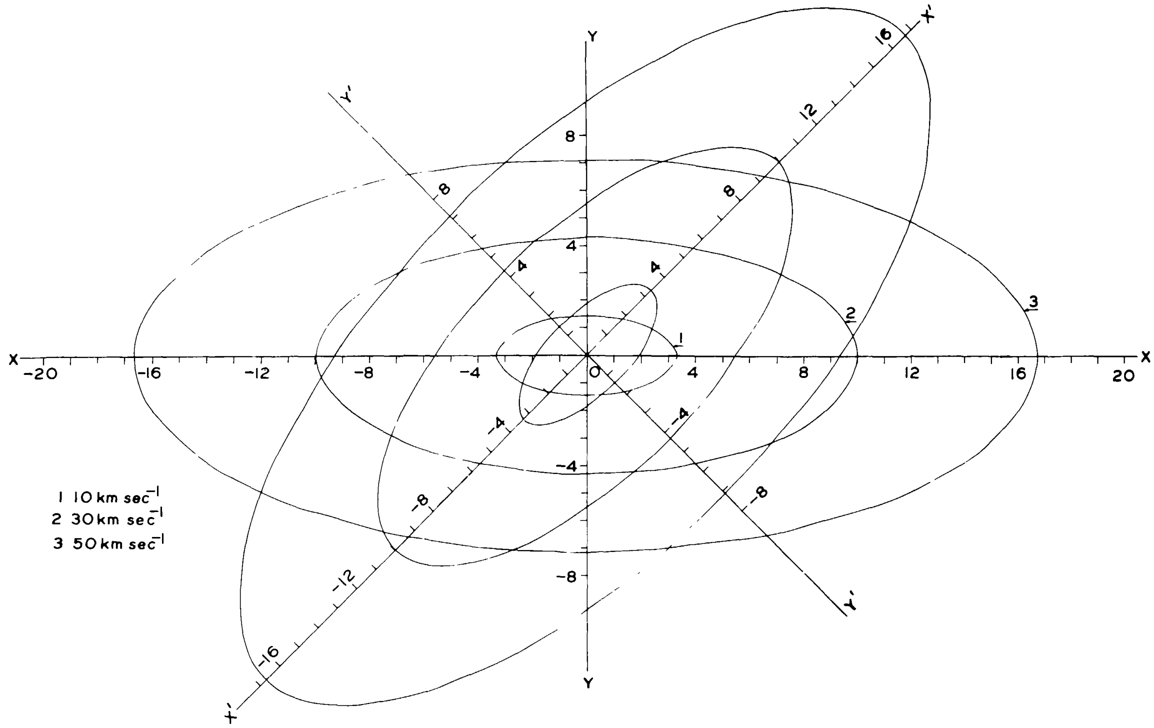

Fig. 1. The iso-velocity contours in the x–y plane for a rotational velocity field in Kippenhahn–Schlüter's model of a quiescent prominence.

Furthermore, the introduction of a velocity field in the x–y plane imposes a density distribution in the y-direction, given by Equation (14).

The iso-velocity contours are shown in Figure 1 and the density variation along the y-axis, which is of an exponential nature, is given in Equation (14).

Appendix

To trace the iso-velocity contours, the following procedure has been taken recourse to, as given by Yefimov (1954). He has shown that for any equation of a line of 2nd order given by

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0,$$

one has to transfer the system to the origin of the ellipse defined by coordinates,

$$x = \tilde{x} + x_0, \quad y = \tilde{y} + y_0 ;$$

which is done through the transformation

$$x_0 = \frac{\begin{vmatrix} B & D \\ C & E \end{vmatrix}}{\begin{vmatrix} A & B \\ B & C \end{vmatrix}}, \qquad y_0 = \frac{\begin{vmatrix} D & A \\ E & B \end{vmatrix}}{\begin{vmatrix} A & B \\ B & C \end{vmatrix}},$$

In our case they happen to be $x_0 = 0$, $y_0 = 0$.

Then, through the transformation,

$$\tilde{x} = x' \cos \alpha + y' \sin \alpha$$

and

$$\tilde{y} = x' \sin \alpha + y' \cos \alpha \,,$$

the inclination of the new axes w.r.t. the old is given by

$$\tan \alpha = \frac{C - A \pm \{(C - A)^2 + 4B^2\}^{1/2}}{2B}$$

The equation of the ellipse is then obtained as

$$\frac{\dfrac{X'^2}{V^2}}{(a_{11} - a_{12})^2} + \frac{\dfrac{Y'^2}{V^2}}{(a_{11} + a_{12})^2} = 1 \,,$$

where the total velocity $\vec{V}$ is represented by

$$V^2 = (v_x^2 + v_y^2)^{1/2} \,.$$

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