GRAVITATIONAL RADIATION AND SPIRALLING TIME OF CLOSE BINARY SYSTEMS (IV)

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Abstract. Binary systems with their primary and secondary component masses less than $2\,M_\odot$ have been investigated to evaluate the rate of emission of gravitational energy (P_B) and spiralling time (τ_0) for them. In all twenty-two binary systems have been considered. It is found that in spite of the same mass range, these systems form two distinct groups. New relations have been given between P_B and τ_0 for each group. For a few eccentric orbit systems the rate of decay of orbital periods due to the loss of energy from the system via gravitational radiation emission has also been given and compared with a short-period binary pulsar.

1. Introduction

Earlier work on the subject (Padalia, 1987, 1988, 1989) has been aimed at finding a relation between P_B and τ_0 for binary systems in typical mass group. In another paper (Padalia, 1988) systems with (B) spectral types were taken for P_B and τ_0 relations. The present work is an extension of the previous work in a more systematic way as the binary systems of the same mass group have been taken with their primary components lying in the mass range between $1\,M_\odot$ and $2\,M_\odot$. Out of the twenty-two binary systems investigated in the present paper, four systems have been found to have eccentric orbits. The rate of decrease of orbital period due to the gravitational radiation emission have been determined for these four (eccentric) systems. It has been found that the rate of decrease of orbital period for these systems is slower than for a binary pulsar PSR 1913 + 16 which is in the same mass group and a fast rotating (pulsar component) highly eccentric and short-period binary system. Table I gives the parameters of the binary systems used in the present paper.

2. Equations Used

The equations for determining P_B and τ_0 relation have already been cited in our earlier paper (Padalia, 1987); viz.,

$$P_B = \left(\frac{\mu}{M_\odot}\right)^2 \left(\frac{M}{M_\odot}\right)^{4/3} P^{-10/3}\, 3.0 \times 10^{26}\ \mathrm{W}\,, \tag{1}$$

$$\tau_0 = \frac{5c^5 a_0^4}{256 G^3 \mu M^2}\,. \tag{2}$$

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TABLE I

Gravitational radiation, spiralling time and rate of change of period ($\dot{P}$) due to gravitational radiation emission of twenty-two binary systems

Name of the binary systems	M_1 ($M_\odot$)	M_2 ($M_\odot$)	Orbital period (in days)	Radius of relative orbit a_0 ($R_\odot$)	Power output (P_B) (W)	Spiral time (τ_0) (years)	X ($\log P - 19$)	Y ($\log \tau_0 - 10$)	Eccentricity of the orbits	Rate of change of period ($\dot{P}$) due to grav. rad. emission
UX Her	1.88	0.49	1.549	7.51	83.4×10^{19}	2.16×10^{11}	1.921	0.334	–	
Y Leo	1.91	0.63	1.686	7.95	101.2×10^{19}	1.94×10^{11}	2.005	0.288	–	
Z Dra	1.82	0.44	1.357	6.77	101.2×10^{19}	1.72×10^{11}	2.005	0.235	–	
AR Lac	1.30	1.29	1.983	9.12	114.4×10^{19}	2.37×10^{11}	2.058	0.375	0.11	-8.85×10^{-15}
V 805 Aql	1.85	1.50	2.408	11.31	138.2×10^{19}	2.62×10^{11}	2.141	0.418	–	
WW Aur	1.81	1.75	2.525	11.91	147.7×10^{19}	2.65×10^{11}	2.169	0.423	–	
RZ Cas	1.81	0.50	1.195	6.26	194.6×10^{19}	1.09×10^{11}	2.289	0.037	–	
FO Vir	1.90	1.28	0.775	4.60	296.1×10^{19}	5.76×10^{10}	2.471	−0.240	–	
V 1073 Cyg	1.37	0.47	0.786	4.390	461.8×10^{19}	4.66×10^{10}	2.664	−0.332	0.12	-1.87×10^{-14}
RW CrB	1.60	0.42	0.726	4.30	618.2×10^{19}	3.73×10^{10}	2.791	−0.428	0.12	-2.65×10^{-14}
VV UMa	1.94	0.45	0.687	4.38	1120.7×10^{19}	2.62×10^{10}	3.049	−0.582	–	
SV Cam	1.15	0.76	0.593	3.68	2129.5×10^{19}	1.634×10^{10}	3.328	−0.787	–	
GO Cyg	1.39	1.17	0.718	4.10	2782.1×10^{19}	1.009×10^{10}	3.444	−0.996	–	
AK Her	1.43	0.43	0.422	2.91	3327.7×10^{19}	9.33×10^{9}	3.522	−1.031	–	
TX Cnc	1.29	0.65	0.383	2.77	8340.5×10^{19}	5.36×10^{9}	3.921	−1.271	–	
TW Cet	1.29	0.67	0.317	2.447	16523.3×10^{19}	3.15×10^{9}	4.218	−1.502	–	
WZ Oph	1.13	1.11	4.184	14.29	5.8×10^{19}	22.09×10^{11}	0.767	1.344	–	
RS CVn	1.42	1.35	4.798	16.80	7.5×10^{19}	21.7×10^{11}	0.876	1.337	–	
WX Cep	1.02	1.02	3.378	12.01	8.7×10^{19}	14.54×10^{11}	0.942	1.162	–	
EK Cep	1.71	1.07	4.428	15.95	8.9×10^{19}	18.94×10^{11}	0.951	1.277	0.11	-2.634×10^{-14}
FL Lyr	1.02	0.93	2.178	8.83	11.2×10^{19}	4.89×10^{11}	1.050	0.689	–	
RX Hya	1.68	0.40	2.282	9.30	13.7×10^{19}	7.98×10^{11}	1.138	0.902	–	

For determination of the rate of decrease of period, the equation

$$\frac{\dot{P}}{P} = - \frac{96G^3\mu M^2}{5c^5a_0^4(1-e^2)^{7/2}}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \tag{3}$$

has been used (cf. Blanchet and Schäfer, 1989) where e is the eccentricity of the binary orbit; P, the period of the system; $\dot{P}$, the rate of change of period due to gravitational radiation and other parameters have the same significance as in Equations (1) and (2).

3. Discussions and Results

Gravitational radiation P_B and spiralling time τ_0 have been determined for twenty-two binary systems with the aid of Equations (1) and (2), respectively. P (in watts) along the X-axis and τ_0 (in years) along the Y-axis have been plotted in Figures 1 and 2. Figure 1

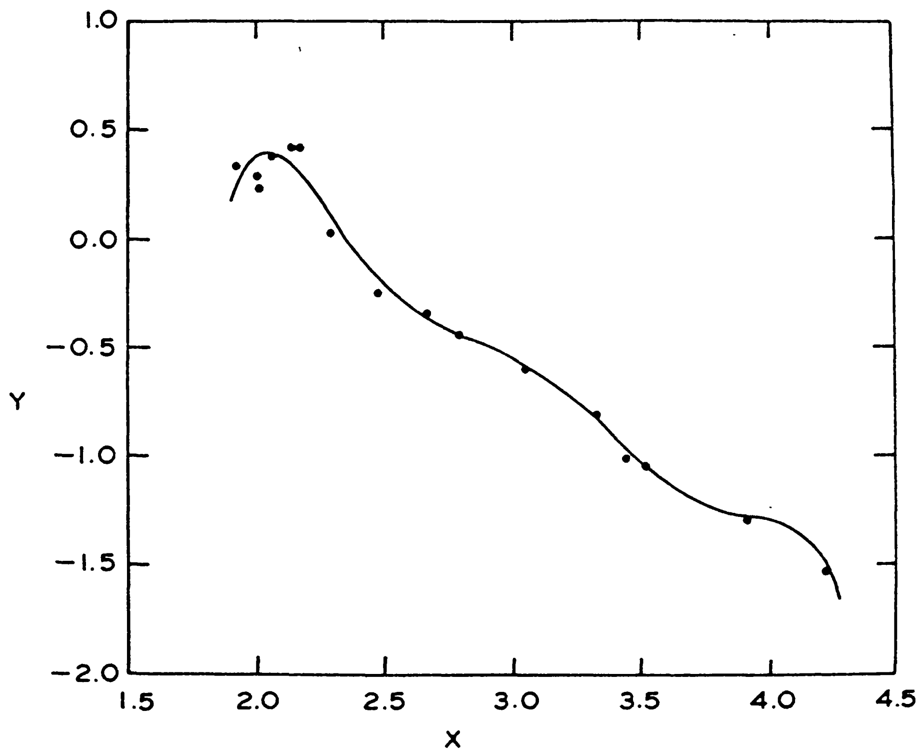

Fig. 1. Relation between gravitational radiation (along X-axis) and spiralling time (along Y-axis) for the first sixteen binary systems given in Table I.

represents first sixteen binary systems of Table I and Figure 2 stands for the remaining six systems of Table I. It is found that the parameters X and Y follow the relations

$$Y = -24.2673x^4 + 2.9986x^3 - 13.4883x^2 + 25.271x - 16.35$$

for 16 binary systems, and

$$Y = 96.7114X^3 - 276.9985X^2 + 259.923X - 78.71$$

for the remaining 6 binary systems, where

$$X = \log P_B - 19 \quad \text{and} \quad Y = \log \tau_0 - 11 .$$

The curves representing these equations are shown by solid lines in Figures 1 and 2, respectively. It is evident from the figures that sixteen systems have spiralling time range 10^9 to 10^{11} years, whereas the remaining six systems have τ_0 of the order of 10^{11} years.

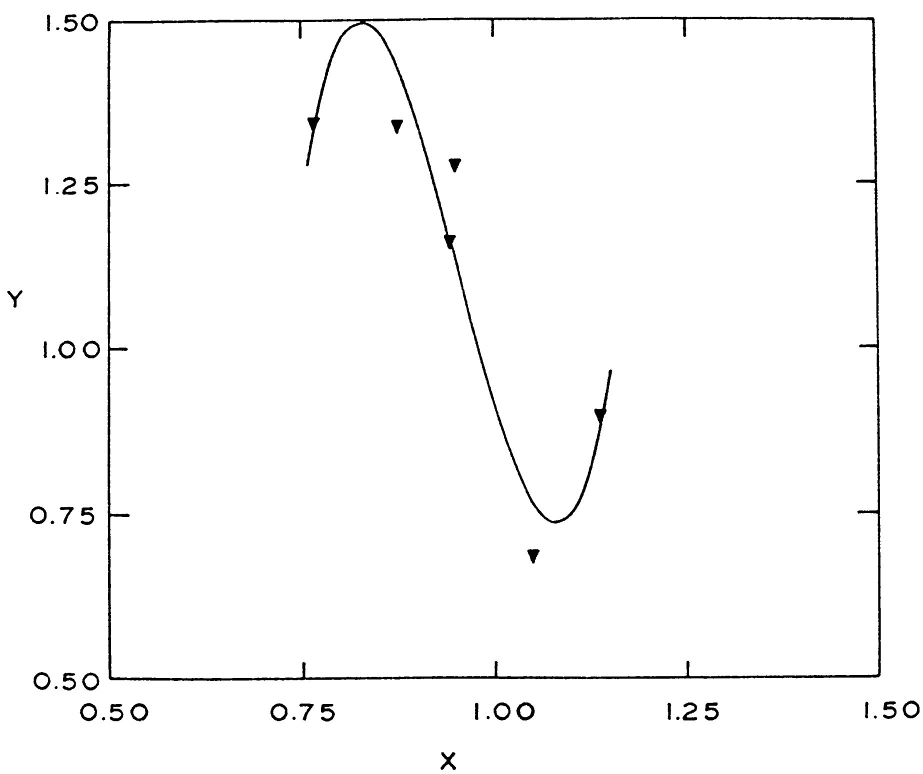

Fig. 2. Relation between gravitational radiation (along X-axis) and spiralling time (along Y-axis) for the last six binary systems listed in Table I.

The values of P_B and τ_0 determined by us are given in Table I. The upper part of the table gives parameters for sixteen systems and the lower part for the remaining six systems.

Peters and Mathews (1963) theoretically computed that the power of gravitational wave emissions should be balanced by a decrease of Newtonian energy of the stars (the rest mass of the components staying constant); hence, (by Kepler's third law) the gravitational wave emission should result in a steady decay of the orbital periods (P) of the stars. The rate of decrease of orbital period due to gravitational wave emission of the four systems (out of he present twenty-two systems) for which eccentricities are known have been given with the aid of Equation (3). The systems of AR Lac ($e = 0.11$), RW CrB ($e = 0.12$), EK Cep ($e = 0.11$), and V 1073 Cyg ($e = 0.12$). The values of $\dot{P}$ are listed in the last column of Table I. It is found that the rate of period decrease ($\dot{P}$) is of the order of 10^{-14}. For RW CrB, $\dot{P}$ is 2.65×10^{-14} which gives a total period change in 100 years (as a large number of binaries like AR Lac are under observations for the last 75 years) – 9.7×10^{-10} days or approximately by 10^{-9} days. The orbital period P of binary pulsar PSR 1913 + 16 has been observed (Taylor and Weisberg, 1982) to be regularly decreasing with

$$\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)_{\text{obs.}} = (-2.40 \pm 0.04) \times 10^{-12} .$$

The theoretical value of $\dot{P}$ due to gravitational radiation loss comes in agreement with the observed one: viz.,

$$\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)_{\text{th.}} = (-2.402 \pm 0.001) \times 10^{-12}$$

(Blanchet and Schäfer, 1989).

The period of this binary pulsar is $P = 7^{\mathrm{h}}45^{\mathrm{m}}$ and eccentricity 0.617. The total period change of this pulsar after 100 years (from the date of its first observations) should be -6.80×10^{-7} days and in six years -4.80×10^{-8} days. Whereas in our present four systems the period decay after 100 years, is of the order of 10^{-9} to 10^{-10} days. Alternatively we can say that the rate of period decrease is 100 to 1000 times slower than for this binary pulsar. It is worthwhile to mention here that the period of a number of binary systems (having short periods) have been determined up to the eighth place of decimal or 10^{-8} days. Therefore, it is easy to detect observationally the effect of gravitational wave emission on the periods of the systems like binary pulsar PSR 1913 + 16 even during a short span of six years as done by Taylor and Weisberg (1982), whereas the period change due to gravitational wave emission of the present binary systems cannot be noted observationally till 1000 years elapse. Therefore, period observations of binary systems with large eccentricities and short period can be helpful to further verify the existence of gravitational radiation and subsequently confirm the general theory of relativity as has been done for the binary pulsar PSR 1913 + 16.

Since the similar massive systems get bifurcated into two groups as per P_B and τ_0 relations (which depend on P and a_0, respectively, of the individual systems) it indicates that the mechanism for binary star formation and evolution may not have been the same for all the twenty-two systems. A possible explanation of having different groups may be given believing that once a star captures another star and forms a bound system (the component initially being at infinite distance and moving with zero-velocity) then the faster rate of loss of gravitational radiation or radiation bursts for a few systems can shrink their orbits abruptly (and consequently forcing the components to revolve faster) as compared to the other systems, thus leading to different periods and separations for a few systems. The faster rate of radiation for a few systems may be due to initial conditions of rotation and revolution of original gas clouds which aspect may play a dominant role. A further study of stars of various mass groups will be more helpful in understanding the details of the mechanism of binary star formation.

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