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On the Analysis of Light Curves in Asteroseismology

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Abstract. We provide a detailed introduction to the main problems arising when analyzing light curves in asteroseismology. Attention is first paid to the signal model delivered by the pulsating stars and to the noise sources corrupting this model in photometric observations. The main pitfalls and ambiguities occurring in Fourier analysis are summarized and illustrated. Some classical, Least Squares (LS) based methods for spectrum analysis are analyzed and commented on from the point of view of ill-posed problems. The insight that can be gained from such analyses is discussed.

Key words. Fourier analysis—noise sources—ill-posed problems.

1. Introduction

Pulsating stars form a subclass of variable stars. The luminosity variations exhibited by these stars depend upon their mass, structure and chemical composition. The purpose of asteroseismolgy is to infer physical parameters for the stars (internal structure, effective temperature, magnetic field, gravity, mass, inclination angle, ...) by analyzing their pulsation patterns. The overall strategy can be divided in to two main parts. In the first part, the data are collected under the form of variability light curves. After a series of operations on these data (reduction, noise analysis, spectrum estimation, see S. Joshi's paper in this issue for examples of roAp stars), the eigenfrequencies are determined, leading to the eigenmodes of pulsation. The second part deals with designing numerical models of stars (and of the corresponding pulsation patterns) by perturbing fluid equations (see e.g., Finley *et al.* (1997) for atmosphere's models). The better the agreement with the data, the better the model (see e.g., Castanheira *et al.* (2004) for a detailed example of a White Dwarf).

The present paper introduces in detail the first part of the above strategy: we present an overview of the techniques and problems related to the determination of the pulsation's frequencies. In section 2, attention is first paid to the mathematical model for signals obtained from pulsating stars; important pitfalls and ambiguities occurring in Fourier analysis are illustrated. We then turn to the noise sources corrupting this model in photometric observations. The third section comments and analyzes two classical methods (CLEAN, SPD) for Fourier Transform (FT) analysis. General Least Squares (LS) based methods are further analyzed and commented from the point of view of ill-posed problems in section 4. We discuss how this point of view can improve the analysis of light curves and lead to different, sometimes more appropriated analysis methods. Section 5 draws some conclusions.

2. Light curves in asteroseismology

2.1 *Signal model, Fourier analysis and data gaps*

Many pulsating stars exhibit small amplitude variability (e.g., sun, rapidly oscillating A peculiar (roAp) stars, white dwarfs, ...). Small amplitude oscillations of spherical objects can be described in terms of spherical harmonics. For a particular mode, the oscillations of scalar quantities such as radial velocity or luminosity can be written in the form: $m(t) = m_0 + a_0 \cos(2\pi \nu_0 t - \phi_0)$, where $m(t)$ and m_0 are respectively the instantaneous and mean scalar quantity (magnitude in the following), a_0, ν_0 and ϕ_0 are respectively the amplitude, frequency and phase of the mode. Several modes may be simultaneously excited in the star, in which case the variable magnitude becomes $m(t) = m_0 + \sum_i a_i \cos(2\pi \nu_i t - \phi_i)$.

For larger (but still periodic) amplitude oscillations, the elementary variability shape may not be sinusoidal anymore (e.g., Cepheids). In this case, peaks appear in the FT at multiples of the fundamental frequency (1 over main period). Hence the signal model is again of the form $m(t) = m_0 + \sum_i a_i \cos(2\pi \nu_i t - \phi_i)$, where a_i, ν_i and ϕ_i are respectively the amplitude, frequency and phase of the harmonics. The respective $\{a_i\}$ and $\{\phi_i\}$ determine the shape of the curve (see e.g., Poretti 2002). In all cases, the variability signal above is observed during a finite time. Assuming that no noise corrupts the light from the star to the detector, the observed magnitude becomes: $s(t) = m(t) \times w(t)$, where $w(t)$ is usually referred to as *observing* or *temporal window*.

The main justification of FT analysis is (in principle) to allow one to isolate the elementary frequencies of the signal. This is useful since several modes/harmonics are in general present in variability curves, resulting in (sometimes very) complicated spectra. Dropping the constant term m_0 in the signal $s(t)$ above, the FT can be written as

$$\begin{aligned} \widehat{s}(\nu) &= \widehat{m}(\nu) \star \widehat{w}(\nu) \\ &= \frac{1}{2} \sum_{i=1}^{K} a_i e^{j(\phi_i - \frac{\pi}{2})} \widehat{w}(\nu - \nu_i) - a_i e^{-j(\phi_i - \frac{\pi}{2})} \widehat{w}(\nu + \nu_i), \end{aligned} \tag{1}$$

where $\widehat{\ }$ and $\star$ denote respectively FT and convolution, $j = \sqrt{-1}$ and $\widehat{w}(\nu)$ is the *spectral window*. Equation (1) shows that we face a deconvolution problem: from $\widehat{s}(\nu)$, we seek to reconstruct $\widehat{m}(\nu)$.

Below are some examples.

(a) The simplest case is that of one single mode (with parameters a_0, ν_0, ϕ_0) observed during one single observation night. Instead of a Dirac delta function, the contribution of the spectral window appears at the frequency $\nu_0 = 1/T_0$. For box-car observing window, $\widehat{w}$ is a sinc function ($\text{sinc}\, x = \sin x/x$). The power spectrum (PS, or periodogram, squared FT modulus) becomes: $|\widehat{s}(\nu)|^2 = \frac{1}{4} T^2 a_0^2 \,\text{sinc}^2[\pi T(\nu - \nu_0)]$, where we assumed that the observation duration T is much greater than the oscillation period T_0. The *frequency resolution* (minimum frequency spacing for two different frequencies to be accurately localized) corresponds basically to the width of the sinc function ($2/T \approx 70\,\mu$Hz for one 8-hour night). Hence, the longer the observation, the better the resolution.

(b) In the case of two oscillations (with parameters $\{a_1, \nu_1, \phi_1\}$ and $\{a_2, \nu_2, \phi_2\}$) observed during one clear night, the PS becomes

$$|\widehat{s}(\nu)|^2 = \frac{1}{4}T^2 \{a_1^2\text{sinc}^2[\pi T(\nu - \nu_1)] + a_2^2\text{sinc}^2[\pi T(\nu - \nu_2)]\}$$
$$+ 2a_1a_2\text{sinc}[\pi T(\nu - \nu_1)]\text{sinc}[\pi T(\nu - \nu_2)]$$
$$\times \cos[\pi T(\nu_2 - \nu_1) - (\phi_2 - \phi_1)]. \quad (2)$$

The PS is not just the superimposition of the two elementary power spectral responses of example (a) because of the third (interference) term. For a given T, this term is important whenever ν_1 and ν_2 are close and T is not sufficiently large w.r.t. the pulsation period. In this case, the phase difference is important as well. This is illustrated in Fig. 1, left panels, with $\nu_1 = 1$ mHz and $\Delta\nu = \nu_1 - \nu_2 = 42\,\mu$Hz. The beating of the two frequencies creates an equivocal representation of the signal. The ambiguity arises in the variability light curve also, which appears as one single cosine amplitude modulated mode even though two closely separated modes are present. The situation of closely separated modes is frequent in asteroseismology (magnetic (Unno *et al.* 1989) and rotational (Ledoux 1951) splitting); the presence of amplitude modulated modes also (see Handler 2004 for a review of this phenomenon in different classes of pulsating stars). In the latter case, the more damped the oscillation, the less narrow the corresponding spectral representation (see Christensen-Dalsgaard (2003), p. 20; Samadi *et al.* (2003) and references therein for more on stellar oscillations' excitation and damping). In both cases unfortunately, the observer has no mean to determinate what is really happening in the star, apart from increasing the resolution. From the PS curves (Fig. 1, left panels), one sees that the maxima are *not* at the actual frequencies. Note also that frequencies only do not tell the whole story: the PS strongly depends on the phases of the oscillations. According to $\Delta\phi = \phi_2 - \phi_1$ in equation (2), the maxima may be moved either inside or outside the actual frequency pair.

(c) A solution for increasing the resolution is to observe the same star during two (or more) consecutive nights. In this case, the PS becomes for a single oscillation $|\widehat{s}(\nu)|^2 = T^2a_0^2\text{sinc}^2[\pi T(\nu - \nu_0)]\cos^2[\pi d(\nu - \nu_0)]$, where d is the time interval between the two observation sessions (typically $d = 24$ h). The sinc^2 spectral window's envelope gets modulated by a cos^2 function, yielding a fine structure. The side lobes ("one-day" aliases) are separated from the main lobe by 11.5 μHz typically ($\pm 1\,d^{-1},\ \pm 2\,d^{-1}$, etc.).

(d) Let us now take a look at the previous case of two oscillations (example (b)), observed during two nights (Fig. 1, right panels). On the one hand, the maxima are closer to the original frequencies than in the one-night case since the interference term's contribution has been reduced (T has been increased). On the other hand, nine main peaks appear now in the PS. The fine structure (one-day aliases) are clearly visible.

The effects described above complicate the frequency representation of the oscillations. They can be far more complicated to understand when $m(t)$ is unknown, and when noise corrupts the data (see Christensen-Dalsgaard & Gough 1982).

2.2 *Noise sources corrupting the model: The case of photometric observations*

The pulsation signal $m(t)$ of section 2.1 suffers first from reddening and absorption when propagating through the interstellar medium. The atmosphere presents *sky*

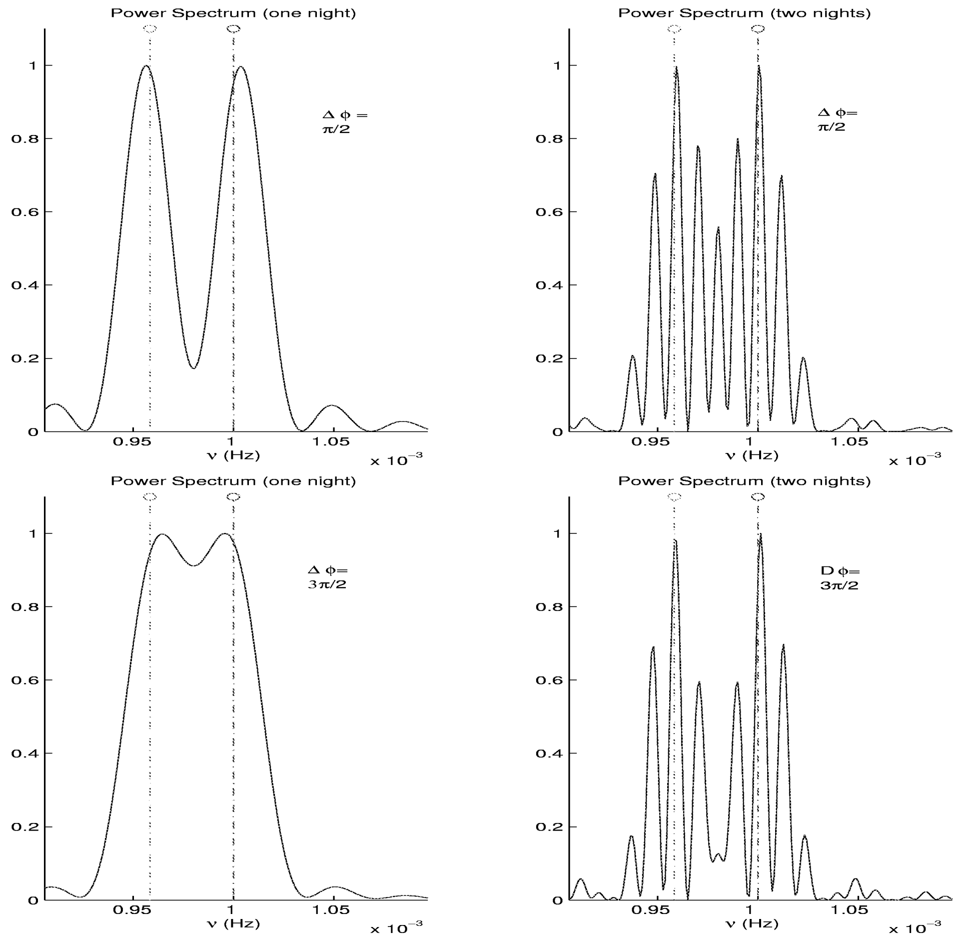

Figure 1. Data with gaps, two oscillations: comparison of the power spectra for one night (left panels) and two nights (right panels). Upper panels: $\Delta\phi = \pi/2$. Lower panels: $\Delta\phi = 3\pi/2$.

transparency variations depending on airmass, humidity, dust, etc.; this yields noise at frequencies less than ≈ 1 or 2 mHz. Atmosphere also produces *scintillation* caused by atmosphere's density fluctuations (Warner 1988), the corresponding noise's energy being at higher frequencies (Dravins *et al.* 1998). The former is usually removed by subtracting a few sinusoids to the data spectrum (*prewhitening method*, Ponman 1981). The latter is often larger than photon noise for relatively (e.g., roAp) bright stars. It can be decreased by using larger apertures (Young 1967) – and by observing from Antarctica, see Fossat's paper in this issue. Indeed, cosmic rays, clouds, planes, meteors, etc. may cause bad data points creating gaps. The signal that eventually falls on the detector is amplified according to the random gain and to the dead time of the photomultipliers (equivalent though less pronounced noises occur with CCDs) and the resulting signal is shaped for digitization. All these operations yield further distortions. Many other insidious noise sources may (and often do) occur: e.g., moisture/dust on optics and filters, telescope tracking oscillations (spurious periods of ≈ 2 to 4 sidereal minutes), moonlight reflection inside the telescope (spurious periods according to the dome rotation), etc. (see Martinez 1993 for an extensive list). Consequently, it must be kept in mind that the signal model of section 2.1 is the very idealization of what the recorded "oscillation signal" may actually look like. The validity of the model may sometimes be highly questionable. On the other hand, the noise list above shows how difficult it can be to describe analytically departures from this model. Indeed, standard reduction techniques help reducing these effects, but to some extent only, and they may lead to artifacts as well (Balona 2002).

The comments above highlight how the simultaneous effects of gaps in the data, beating between close frequencies, damping and various noises can be confusing and lead to erroneous conclusions in the frequency analysis. If one goes for FT analysis, phase information is essential and amplitude rather than power spectra must be preferentially investigated. In order to increase the frequency resolution/reduce the data gaps, astronomers are often using multi-site, worldwide campaigns (Whole Earth Telescope, WET: e.g., Nather *et al.* 1990; STEllar PHotometry International: e.g., Michel *et al.* 1992; Delta Scuti Network: e.g., Breger *et al.* 1995). Because uncooperative weather, atmospheric and instrumental noises are unavoidable, the pitfalls and noises summarized in sections 2.1 and 2.2 remain major limiting factors for the accuracy of the variability analysis. As such, their nature and effects deserve to be as precisely understood as possible.

3. FT-based methods

Several attempts have been made to analyze noisy Fourier spectra in some automated and clever way. Two of them are discussed here: CLEAN is widely used — but often as a "black-box" unfortunately, as confessed by experienced asteroseismlogists. The Several Peaks Deconvolution (SPD) method may be considered as a "one-shot" variant of CLEAN.

3.1 *The CLEAN method*

The CLEAN method was conceived by Jan Hogbom (1974) for eliminating side lobes in radio interferometry. Schwarz (1978) showed that CLEAN is equivalent to LS fitting. CLEAN assumes as signal model $s(t) = \sum_{i=1}^{K} a_i \sin(2\pi \nu_i t + \phi_i) \times w(t)$, so that the associated FT is as in equation (1). The algorithm performs the simple iterations (in the sequel, the symbol $\widetilde{}$ denotes estimates):

(1) Find the frequency $\widetilde{\nu}_i$, amplitude $\widetilde{a}_i$ and phase $\widetilde{\phi}_i$ of the largest peak,
(2) Subtract the scaled contribution of $\widehat{w}(\nu - \widetilde{\nu}_i)$ from the FT (1),
(3) Back to 1, or stop iterating as the FT residual is inferior to the noise level.

An example of CLEAN for a simulated oscillation signal is presented in Fig. 2 ($K = 5$). In this case, amplitudes, frequencies and phases are found with a relative precision of 10^{-3}. Note however that, even without noise, the residual is not zero. Indeed, at each iteration the maximum peak's frequency $\widetilde{\nu}_i = \nu_i + \delta\nu_i$ is displaced from the true frequency ν_i because of interferences. The estimated amplitude and phases are slightly incorrect as well (say $a_i + \delta a_i$ and $\phi_i + \delta\phi_i$). Hence, terms of the form $(a + \delta a_i)_i e^{j(\phi + \delta\phi_{i_i} - \frac{\pi}{2})} \widehat{w}(\nu - \nu_i + \delta\nu_i)$ are subtracted from the FT of equation (1), so that one cannot hope the sum in equation (1) to be perfectly dismantled. The algorithm is biased. Moreover, if we continue iterating in Fig. 2, an artifact peak will be

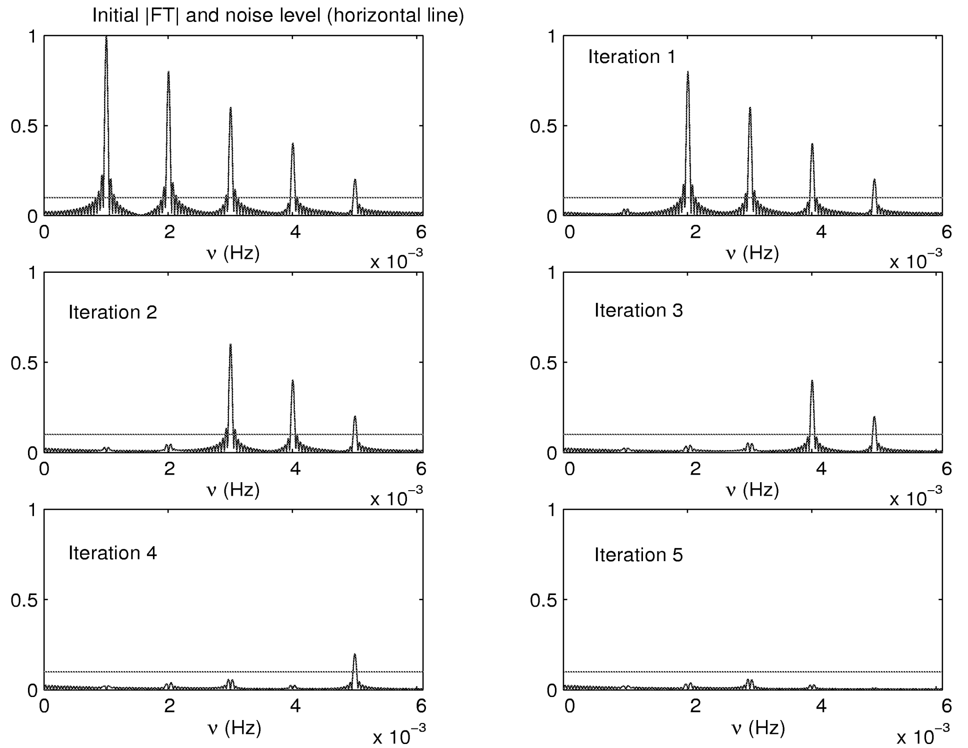

Figure 2. CLEAN's results for a 5 sinusoids (frequencies at 1, 2, 3, 4 and 5 mHz) signal.

found somewhere around 3 mHz, and then further ones if we continue iterating. For real signals exhibiting closely separated modes and/or gaps and/or noise, CLEAN's drawbacks are emphasized. Foster (1995) proposed an improved version of CLEAN (CLEANEST) which suffers however from the same impairments when there are gaps (Janot-Pacheco *et al.* 1999). Note also that a proper noise level estimation is capital. Many techniques exist for that purpose, see e.g., Roques *et al.* (1999) and references therein. In the particular case of WET data, Castanheira *et al.* (2004) use the fact that FT peaks whose amplitude is above four times the square root of the average power has a 1 over 1000 probability to be noise. Once high amplitude peaks have been removed, the noise in the residual FT can be estimated as the square root of the average power (Kepler 1993). For WET data, it is then useful to weight differently several runs: the higher the signal to noise ratio of the run, the larger the corresponding weight (Handler *et al.* 2002).

3.2 *Several Peaks Deconvolution (SPD)*

Since the reciprocal peaks' interferences contribute to CLEAN's bias, one may seek to estimate the set of $\{a_i, \phi\}$ jointly. This was proposed by Pfeiffer (1993). The basic idea is as follows:

(1) Determine a number of K main modes and associated frequencies $\{\widetilde{\nu}_i\}$,
(2) Compute *jointly* the $\{\widetilde{a}_i, \widetilde{\phi}_i\}$ ("one-shot" (instead of iterative) approach),
(3) Repeat 1 and 2 for other sets of $\{\widetilde{\nu}_i\}$ to find the best fit w.r.t. to the data.

The signal model is unchanged, and the FT of the data is again modeled as in equation (1), in which only positive frequencies are considered (say N frequency points). For every frequency $\nu = 0, \dots, \nu_{\max}, \widehat{s}(\nu)$ equals the sum of K contributions. The goal is to find the $\{z_i\} = \frac{a_i}{2} e^{j(\phi_i - \frac{\pi}{2})}$ which resolve this system of equations. The amplitudes and phases can be obtained by $a_i = 2|z_i|$ and $\phi_i = \arg\{z_i\} + \frac{\pi}{2}$. This problem can easily be put in matrix form: $\widehat{W} Z = \widehat{S}$, where $Z = [z_1\ z_2 \cdots z_K]^T$, $\widehat{S} = [\widehat{s}(0) \cdots \widehat{s}(\nu_{\max})]^T$, and $\widehat{W}$ is the $(N \times K)$ *observation matrix*. The system being overdetermined (more data points N than peaks K), one may obtain an approximated solution by LS. The formal solution Z_0 is

$$Z_0 = \arg\min_Z \|\widehat{S} - \widehat{W} Z\|^2 \Rightarrow Z_0 = (\widehat{W}\widehat{W}^H)^{-1}\widehat{W}^H\widehat{S}, \tag{3}$$

where ^{H} denotes Hermitian transposition. As for CLEAN, the best LS fit of SPD may not lead to good solutions for noisy signals. Why?

4. The point of view of ill-posed problems

Usually, the determination of the oscillations' parameters is made by LS[1] fitting, either in the frequency domain (see above), or in the temporal domain[2] (prewhitening, close to CLEAN, see Ponman 1981). In both cases, the problem is equivalent to equation (3). The solution minimizes the MSE, but how reliable is this solution? In other words, how close are the estimated parameters from the actual ones?

That an LS fit (or any other data fitting technique) may not be meaningful is something that any astronomer is probably aware of[3], and the precautions regarding the use of LS have been discussed elsewhere (see Isobe *et al.* (1990) for applications in Astronomy). It is maybe worth illustrating differently the meaning and implications of the questions above. Let us take for that purpose the simple example of a 2×2 observation matrix $\widehat{W}$, and a 2 points data vector $\widehat{S}$ for the system $\widehat{W} Z = \widehat{S}$. In this case the solution Z_0 is the intersection of two straight lines, see Fig. 3.

The sensitivity to noise can be more generally formulated as follows. Denote by λ_i (resp. v_i) the singular values (resp. vectors) of $\widehat{W}$, then the LS solution of equation (3) becomes $Z_0 = \sum_{i=1}^{K} \frac{1}{\lambda_i^2}(v_i^T \widehat{W}^T \widehat{S}) v_i$. We can always write the data vector as $\widehat{S} = \widehat{S}_{\text{star}} + \widehat{S}_{\text{noise}}$, where $\widehat{S}_{\text{star}}$ is the signal delivered by the star and $\widehat{S}_{\text{noise}}$ is the contribution of all the possible noise sources of section 2.2. Then we have

$$Z_0 = \sum_{i=1}^{K} \frac{1}{\lambda_i^2}(v_i^T \widehat{W}^T \widehat{S}_{\text{star}}) v_i + \sum_{i=1}^{K} \frac{1}{\lambda_i^2}(v_i^T \widehat{W}^T \widehat{S}_{\text{noise}}) v_i. \tag{4}$$

The second term in (4) shows that for small λ_i, the reconstructed LS solution suffers from *noise amplification*. Furthermore, denoting by δX a perturbation on a vector X

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[1]Invented by Gauss and Laplace in the beginning of the XIXth century for astronomical applications.

[2]Combination of linear and non-linear LS are sometimes used, see e.g., Kurz *et al.* (1997) for a detailed example.

[3]A major justification of LS is to yield the same estimate as the Maximum Likelihood Estimate for Gaussian residuals — a condition not often met in practice.

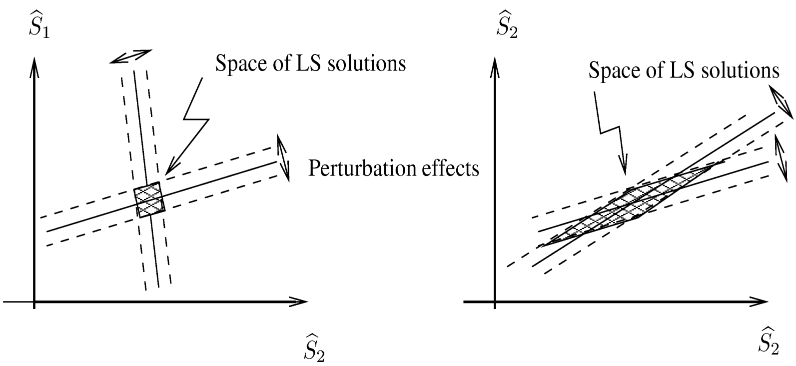

Figure 3. Illustration of the instability phenomenon in ill-conditioned problems. Left: The LS solution Z_0 is at the intersection of the two solid lines. If some perturbations occur on this setting (noise in the data), the resulting LS solutions will be limited to the dashed zone. Right: $\widehat{W}$ has been changed. The LS solution Z_0 is still at the intersection of the two solid lines. With the same perturbation power as in the left panel, the corresponding LS solutions are very unstable (S. Jankov 2003, personal communication).

and by $\|X\|$ its norm, one can show that $(\delta Z_0)/\|Z_0\| \leq (\lambda_{\max}/\lambda_{\min})(\delta\widehat{S}/\|\widehat{S}\|)$. The ratio $\lambda_{\max}/\lambda_{\min}$ is usually referred to as the *condition number* of the matrix $\widehat{W}$. The former inequality shows that little perturbations on the data create space for strongly different LS solutions. In asteroseismology, one obtains typically good conditioning numbers for one (clear) night ($\lambda_{\max}/\lambda_{\min} \approx 1$ or 2), whereas this number becomes very large for two nights ($\lambda_{\max}/\lambda_{\min} \approx$ a few hundreds). The effects of gaps are to introduce zeros in the $\widehat{W}$ matrix whose columns tend to become similar. The weakest singular values tend to be even weaker, leading to the noise amplification in (4) and to a situation comparable to that illustrated in Fig. 3, right panel. The solutions $\{\nu_i, a_i, \phi_i\}$ obtained in this case are not trustworthy: because of noise and gaps, the data are just too different from the model, which corresponds to good observation conditions. Note that the analysis above does not leave us with doubt only: we have some mean to measure in a precise, mathematical form, however far our results may be from the actual parameters.

This kind of problems are named *ill-posed* or *ill-conditioned* problems, which means that the solution to a proposed problem may not exist, may not be unique, and may not be stable. In this case, many remedies exist, whose roots lie in the analyses above. Improved LS-like methods have been on the one hand designed a long time ago: e.g., reweighted LS allowing one to weight the data points/sets according to some confidence criterion; total LS which accounts for the presence of noise both in the data and the observation matrix (Golub & Van Loan 1980); different periodograms may also be used, e.g., Lomb-Scargle periodogram (Scargle 1982); L_1 norm in equation (3) can be minimized (which increases the robustness); and combinations thereof (see Branham (1990) for a review). Starting with Tikhonov & Arsenin (1977) on the other hand, a series of signal restoration methods have been conceived. For example, one may truncate the singular value decomposition so that the reconstructed solution in equation (4) involves only sufficiently large eigenvalues; *a priori* constraints on the reconstructed solutions (e.g., no aliases allowed, or smooth spectra) can be imposed (*regularization* solutions in Bayesian approaches), etc. Such deconvolution problems constitute an active field of research in the signal processing community (see the IEEE literature) and, to a less extent, in asteroseismology (see e.g., Roques *et al.* 1999). In many approaches, the deconvolution is often performed over restricted frequency supports only (*i.e.*, where signal is large w.r.t. noise) so that the conditioning is better. But one understands that in this case the question becomes: how to separate signal and noise subspaces? Indeed, we miss knowledge about the part of the spectrum in which the pulsations may occur, since this is precisely what we are looking for. An important branch of these developments is supported by time-frequency representations, the most widely used being Wavelets (Daubechies 1992) and Matching Pursuit (Mallat & Zhang 1993). In these representations the time information is not lost (as in Fourier spectra). This feature is particularly attractive for noise discrimination (see Donoho & Johnstone 1994) and amplitude modulation detection. Roques *et al.* (1999) have used MP to detect which frequency intervals contain signal, and to perform deconvolution on the corresponding frequency supports. Such approaches allow both a high resolution restoration of $m(t)$ – even better than FT on some frequency intervals – along with a precise evaluation of the reliability of the solution.

5. Conclusions

Because uncooperative weather, atmospheric and instrumental noises are unavoidable in astronomical observations, data gaps and other noise sources remain major limiting factors for the accuracy of frequencies determination in asteroseismology. As such, their effects deserve to be most precisely understood. When analyzing light curves, astronomers traditionally use LS techniques and FT representation. Being aware of CLEAN-like method's impairments, the most careful ones use them for guidance only. In practice, the FT is often dismantled by hand, and it is analyzed according to the available information about the observational conditions, to the astronomers' previous knowledge of the star and, last but not least, to their own experience. In the parlance of Bayes, this is named *a priori* information. Indeed, such traditional methods often work, otherwise asteroseismology would not have shown so many excellent results – those reported in this special issue for example. However, many other efficient signal analysis tools exist – and many remain to be developed. These are interesting, regarding at least three points.

Firstly, these tools can provide a precise evaluation of the confidence level of the results.

Secondly, they may be more appropriate than the traditional techniques discussed above in the case of particular asteroseismologic phenomena (e.g., wavelets/MP help analysing amplitude modulation and frequency drifts).

Thirdly, such approaches provide an effort towards an explicit, mathematical formulation of the *a priori* information that can be injected in the analysis — in contrast to the more implicit *savoir-faire* of experienced asteroseismologists.

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