

On the Differential Rotation of Stars

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Abstract—We discuss methods for analyzing the observational manifestations of differential rotation (DR). Based on the literary sources and our estimates (for 4 cool dwarfs), a list containing 75 stars was compiled. Using an example of analysis of the ‘ $\Delta\Omega-T_{\text{eff}}$ ’, ‘ $\Delta\Omega$ –rotation period’ and ‘ $\Delta\Omega$ –Rossby number’ diagrams we compared the determined parameters of DR stars with the results of theoretical studies. Particular attention is paid to the problems of measurements of DR parameters in low-mass dwarf stars, including the completely convective ones. We analyzed the measured parameters of solar-type DR stars. The objects having anti-solar DR ($\alpha < 0$), and possible methods allowing to determine the sign of α are considered. We distinguish the areas of research that in the future may expand our understanding of DR manifestations: a study of DR in the inner regions of stars and an analysis of manifestations of the rotational brightness modulation caused by the spottedness of components in eclipsing-variable systems (subsynchronized systems).

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1. INTRODUCTION

Manifestations of differential rotation (DR) in late spectral class stars are in many respects similar to those found in the Sun, the rotation periods of objects P , determined by the modulation of light curves owing to the presence of spots on the surface of a star depend on the position of these spots in latitude. When the position of the spot changes from the equator to the pole, the value of P as a rule increases. The phenomenon of DR stars itself (caused by the interaction of rotation and convection) is closely related to their magnetic activity and serves as the subject of numerous theoretical studies. DR of the Sun is directly observable and is the most well-studied. For other stars, it can be observationally established during the photometric and spectral measurements. Relatively recently, asteroseismology and helioseismology have opened up opportunities of DR study inside the stars and the Sun. A survey of theoretical research can be found, for example, in [1]. It is worthwhile to particularly mention the papers [2] and [3], the results of which we will often use in the present writing.

It is generally accepted that the DR law is described by an equation of the form $\Omega(\theta) = \Omega_{\text{eq}}(1 - \alpha \sin^2 \theta)$, where $\Omega(\theta)$ is the angular velocity at the latitude θ , Ω_{eq} is the angular velocity at the equator, $\Delta\Omega = \alpha \Omega_{\text{eq}}$. For the Sun $\alpha_{\odot} = 0.2$.

Observational manifestations of DR in the stars can be investigated within several methods. First of all, spectral methods can reveal DR during the Doppler mapping of stellar surfaces. The parameter of differential rotation of a star $\Delta\Omega$ can be found after the cross-correlation analysis of the constructed maps. Examples of such an analysis are presented in [4]. In addition, the DR value can directly be included in the Doppler mapping procedure and be determined while minimizing the deviations of the observed and theoretical profiles of spectral lines (sets of lines). As a result of the analysis the most optimal pair of values is the rotation period and the DR parameter (for an example, see [5]). Another method determining DR from spectral observations is associated with the analysis of Fourier transforms of line profiles in the spectra of rapidly rotating stars. In contrast to the Doppler mapping methods related with the study of line profile variability, this method, engaging Fourier

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analysis can only be applied to the profiles of inactive stars (not containing any features that form in the cool areas on the stellar surface). This method was developed in the study [6] and presented in a series of articles (see [7]). The essential advantage of the method is that the DR parameter can be measured from single exposures of spectral observations, in contrast to the Doppler mapping method requiring a series of spectral observations sufficiently uniformly covering the rotation period of the star.

Finally, the authors of [8] and [9] determined the parameters of DR for representative samples with the number of stars comprising tens of thousands based on the difference in periods, found from the splitting of peaks in the brightness variability power spectra of spotted stars. In the next section, we will look at these studies in more detail.

2. LITERARY DATA SOURCES ON DR STARS

We adopted the data on the parameters of DR stars for comparing them with the results of theoretical studies from a number of literary sources.

Reinhold, Reiners and Basri [8], and Reinhold with Gizon [9] using the Kepler space telescope observations performed an analysis of the light curves of 18 616 and 12 300 objects, respectively, and found their DR parameters. If in [8] the analysis was carried out only based on the data of observations of a limited time interval Q3, then in [9] a more extensive sample Q1–Q14 was considered. These are the most numerous homogeneous data of α and $\Delta\Omega$ determinations, allowing for their statistical analysis. Further, comparing with other literary sources, we will use the data from [9]. They characterize the variations in DR parameters in a wide range of effective temperatures—from 3200 K to 8000 K. Note that the data from [9] has a large spread of values (for example, of the $\Delta\Omega$ parameter) at a fixed temperature. It is not clear at that whether the spread is physical in nature, or as the authors of [5] believe, it may indicate the presence of systematic errors inherent to the method used for determining the parameters in [9].

In addition, we used the DR parameter data for the stars of spectral classes GKM from the compilation work of Kővári et al. [10]. The authors discussed the highly accurate data on α and $\Delta\Omega$, obtained only based on the research using the Doppler and Zeeman-Doppler mapping. One of the main objectives of the study was to establish the differences in DR parameters for single stars and binary system components.

Numerous data on the parameters of DR stars of late spectral classes are contained in a series of papers by Barnes et al. (see [5] and references therein). We have compiled a general list of objects with DR parameters, based on the values given in [11] and in the papers listed in [12]. In addition, the results of our research on the activity of cool dwarfs GJ 1234 [13], KIC 1572802 [14], LHS 6351 [15] and K2-25 [16] have been added to the general list.

A recent study [17] presents the results of analysis of young solar-type stars (aged 120–650 Myr). The masses of stars studied lie in the range of $0.9\text{--}0.95M_{\odot}$, while their periods—in the interval of 0.326–10.6 days. For 15 stars, the presence of magnetic fields of about 8–195 G is established. We used the estimates of DR parameters obtained when constructing the Zeeman-Doppler maps.

The authors of [18] determined the DR parameters for 2562 stars of KA spectral classes. The method of analysis used was based on the frequency analysis and similar to that used in [9]. Unfortunately, the authors of [18] did not give any results of determination of DR parameters for individual objects. We were not able to include this data in our study and were confined only by a comparison with the conclusions given in the paper.

Therefore, when comparing the results of observations with theoretical calculations, we use an integrated list of objects containing 75 stars, compiled on the basis of literary sources and our estimates for 4 cool dwarfs.

3. A COMPARISON OF DR PARAMETERS OF STARS WITH THE RESULTS OF THEORETICAL RESEARCH

Figure 1a shows the $\Delta\Omega$ dependence on the effective temperature according to the data from all the literary sources we found.

In a number of previous studies (see [2]) it was found that the relationship between these two parameters is the most pronounced. Barnes et al. [11] have shown for a limited set of data that $\Delta\Omega$ is proportional to the value of T_{eff} raised to the 8.92-th power (with an error of 0.31). This empirical dependence was confirmed by the results of the research [12], in which a dependence with the temperature proportionality raised to the power of 8.6 was proposed.

The modelling results from [2] indicated two theoretical dependences of $\Delta\Omega$ on temperature: 1) proportional to T_{eff} raised to the second power for stars with $T_{\text{eff}} < 5000$ K and 2) raised to the power of 20

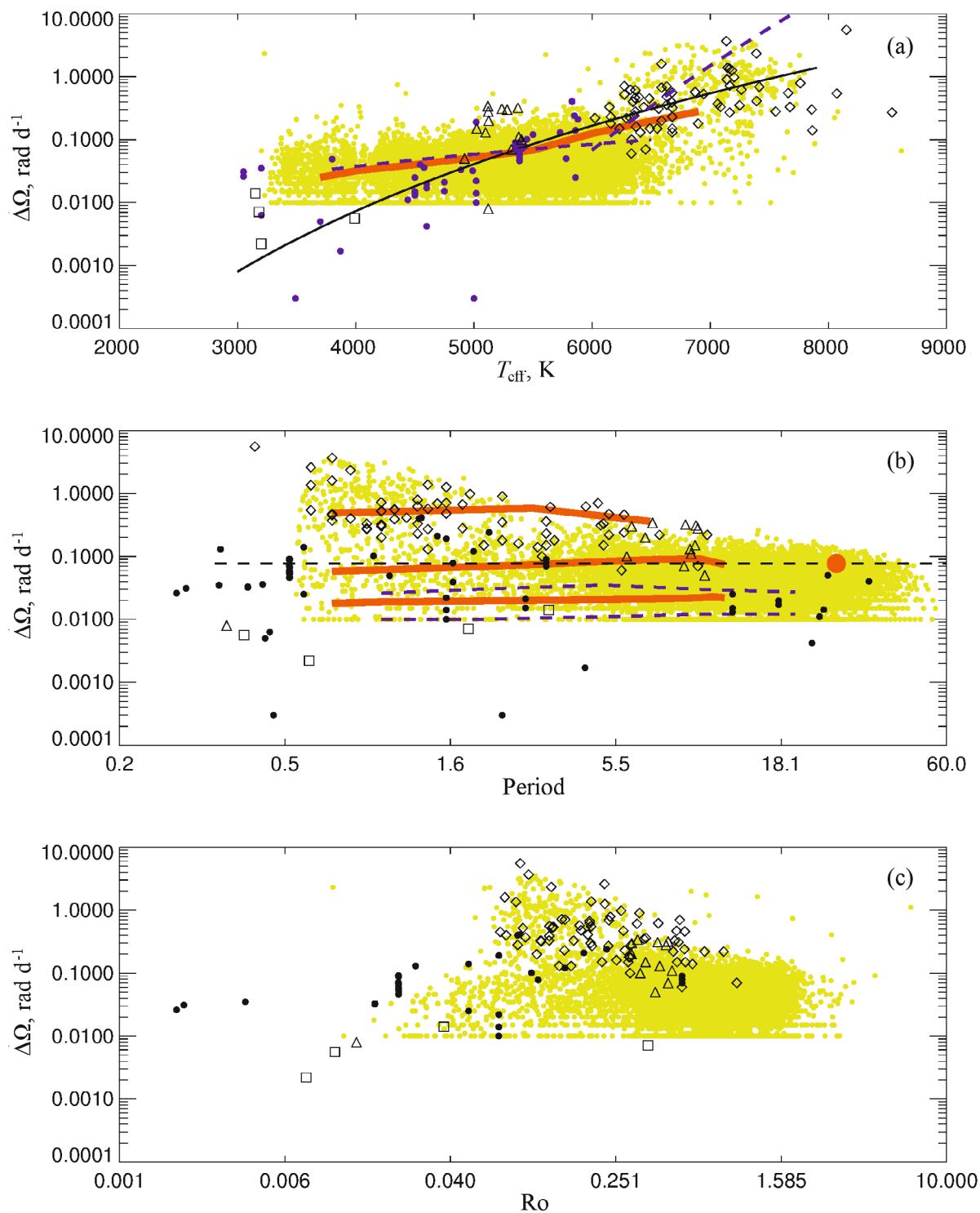


Fig. 1. (a) The diagram of variations in the $\Delta\Omega$ parameter with temperature. A solid thin line describes the empirical dependence [5]. The calculation results from [3] are represented by a thick red line, the results of modeling from [2]—by the dashed lines (see the text for explanations). (b) The diagram of variations in the $\Delta\Omega$ parameter with period. A large red circle denotes the position of the Sun, a thin horizontal dashed line corresponds to the solar $\Delta\Omega$. The calculation results from [3] for stars with masses of 1.275 , 1 and $0.5M_{\odot}$ (from top to bottom) are presented by a thick red line, the simulation results [2] for stars with masses of 0.5 and $0.3M_{\odot}$ (from top to bottom)—by the dashed lines (see the text for explanations). (c) The diagram of variations in the $\Delta\Omega$ parameter with the Rossby number Ro . On all the diagrams, the bright circles denote the stars from [9], the dark circles—the generalized list data, the diamonds—the data from [7], squares—our data for cool dwarfs GJ 1234, KIC 1572802, LHS 6351 and K2-25, the triangles—the data from [17].

for stars with $T_{\text{eff}} > 6000$ K. In general, their combination agrees well with $\Delta\Omega$ values found in [9]. These dependences reflect a break in the data from [2], but are less consistent with the other results (for example, from the integrated list we are considering) for which there is no break. In addition, the results of [2] predict the further growth of $\Delta\Omega$ with increasing temperature (the T_{eff} interval of 7000–7800 K), while in the observational data it is missing. For the objects with $T_{\text{eff}} < 5000$ K the agreement of results from [2] with the observational data is worse than, for example, for the empirical dependence with the indicator of 8.9.

The calculation results from [3] (for the stars aged 1 Byr and rotation periods of 10 days) are presented in Fig. 1a by a thick light line. For the objects with $4000 \leq T_{\text{eff}} \leq 7000$ K the agreement of theoretical estimates (including among themselves) and observational data is quite good. There is not much data about $\Delta\Omega$ for the stars with $T_{\text{eff}} < 4000$ K in our integrated list, and moreover, there is a large scatter of values for them.

Two circumstances make themselves conspicuous when comparing the theory and observational data. For stars with $T_{\text{eff}} > 5500$ K theoretical dependences predict excessive values of the $\Delta\Omega$ parameter as compared to the average for the previously established ones in the study [9] (in Fig. 1a, the curves pass close to the upper envelope of the data array from [9]). At the same time, almost all the measured values of $\Delta\Omega$ of our integrated list are in agreement with the theoretical curves.

In the region of temperatures below 5500 K, taking into account the absence of data from [9] for the objects with $\Delta\Omega < 0.01$ (DR detection limit in [9]) and a larger spread of points compared to the data for hotter stars, the considered dependences (with the exception of the empirical dependence, proportional to T_{eff} to the power of 3.8) describe the upper boundary of the $\Delta\Omega$ values. In addition, it follows from Fig. 1a that the empirical dependence, proposed by the authors of [5], $\Delta\Omega = (0.045 \pm 0.013)(T_{\text{eff}}/5000)^{3.8 \pm 0.7}$, characterizing the objects with $T_{\text{eff}} < 5000$ K, can be used to represent the data of our larger sample in a broader temperature range.

Similar results can be obtained from the consideration of data from [18] for the estimates of DR parameters in 2562 stars of KA spectral classes. For example, according to Fig. 3 from [18], the value of the $\Delta\Omega$ parameter for stars with T_{eff} from 3800 K to 6800 K varies from 0.008 to 0.7–0.8. This is approximately in the same range as in Fig. 1a from our research. The authors of [18] proposed an analytical relationship representing the value of the $\Delta\Omega$ param-

eter as a nonlinear function of T_{eff} and the rotation period.

An interesting feature of the data presented in Fig. 3 of [18] is the presence of stars in the temperature range of 6000–10 000 K, for which the $\Delta\Omega$ parameter can reach 0.6 and higher. We believe that one should treat with caution the $\Delta\Omega$ determinations from [18] for stars of spectral class A and hotter only based on the analysis of their power spectra. Hence, for example, the [19] analysis data led to finding in the AB star power spectra of a specific frequency set, including a separate isolated peak and an array of partially resolved peaks smaller in amplitude. According to the interpretation from [19] the array of peaks is due to the presence of short-lived spots on the surface of a differentially rotating star, and an isolated peak is related to the reflection of radiation from a Jupiter-like planet revolving around a star. A recent study [20] allowed for the first time to interpret the indicated details on the power spectra as the manifestations of r-mode oscillations caused by the presence of Rossby waves. Analogous manifestations of oscillations were established not only for the rapidly rotating A and B stars, but for the Be and γ Dor-type stars as well. Given the results of [20], it is obvious that the question of the DR parameters in stars with $T_{\text{eff}} = 6000$ –10 000 K requires further research.

In our opinion, the diagram illustrating the dependence of the $\Delta\Omega$ parameter on the rotation period of objects is not less informative. The data for the main array of stars are located within the range of $\Delta\Omega$ from 0.01 (detection limit in [9]) up to 0.12. Theoretical calculations lead to the conclusion on a weak dependence of $\Delta\Omega$ on P for a fixed stellar mass (solid and dashed lines in Fig. 1b, constructed according to the results of [3] and [2], respectively). Taking into consideration this weak dependence, the calculations of [3] for a star with a mass of one M_{\odot} agree fairly well with the $\Delta\Omega$ value for the Sun (a large bright circle in Fig. 1b). The region of large $\Delta\Omega$ values (over 0.11) presents objects from [7] and hot ($T_{\text{eff}} > 6500$ K) objects from [9], whose masses exceed the mass of the Sun. Their position agrees well with the predictions of the theory from [3] (a solid line for a mass of $1.275 M_{\odot}$).

Comparing the results of calculations from [2] and [3] for an object with a mass of $0.5 M_{\odot}$ (solid and dashed lines in Fig. 1b) it can be concluded that the types of dependences coincide. However, the dependences themselves have a systematic difference in magnitude of about 0.03 radians a day.

Calculations from [2] for a star with a mass of $0.3M_{\odot}$ (the bottom dashed line) correspond to the

$\Delta\Omega$ parameter of about 0.010 and coincide with the limiting value of this parameter, obtained by the technique from [9].

Determination of DR parameters for low-mass cool dwarfs (including the fully convective dwarfs) is subject to special attention. According to the theoretical predictions from [21] and [22], for stars with axisymmetric dipole fields it should be expected that such objects have a weak DR, in contrast to the noticeable manifestation of DR that can be detected in stars with multipolar fields. A more detailed discussion of the problem can be found in [5], its observational aspects—in the papers of Morin et al. (e.g. [23] and [24]), and theoretical aspects—in [22]. The research of [24] led to establish two groups of M-dwarfs that differ in terms of magnetic properties, though having the same masses and rotation periods, which allowed the authors [24] to suggest two modes of the dynamo mechanism. Other explanations of a coexistence in the completely convective dwarfs of large-scale and small-scale magnetic fields can be found in [22]. In addition, it follows from the results of the listed studies that the theory predicts such α parameter values that are about an order larger in magnitude than those obtained from the observations. A possible reason for such discrepancies may be related with the following circumstance. The M-dwarf magnetic field research (see [24]) is based on the procedure of magnetic field parameter recovery only based on circular polarization, while the temperature mapping was not performed. There are assumptions in [5] that in this case, the $\Delta\Omega$ estimates only based on the V parameter can contain systematic errors.

Finally, the diagram of Fig. 1c shows a comparison of the $\Delta\Omega$ parameter with the Rossby numbers for the studied objects. If we do not consider the objects from [7] and hot ($T_{\text{eff}} > 6500$ K) objects from [9], whose masses exceed the mass of the Sun and for which the $\Delta\Omega$ parameter exceeds 0.11, then for the remaining stars, including the objects of our generalized list, we can assume a decrease in the $\Delta\Omega$ parameter with a reduction of the Rossby number Ro . We have earlier come to a similar conclusion [15] based on a sample which is more limited by the number of objects.

4. SOLAR-TYPE STAR DR

Special attention is drawn to the study of DR in solar-type stars. It is of interest in terms of establishing the past, present and future in the evolution of activity of the star nearest to us, and serves to answer the question how the activity of the Sun could vary over time. In addition, we can make comparisons of

the data for the Sun with the properties of younger and older stars with masses close to $1M_{\odot}$.

As in [25], from all the objects of [9] we have chosen stars with effective temperatures T_{eff} from 5700 K to 5800 K and the acceleration of gravity $\log g$ from 4.4 to 4.5. Both ranges of parameters include solar values.

For the selected stars, we analyzed the properties of active regions (cool spots) on their surface. The spottedness S of the stellar surfaces was determined from the photometric observations using the technique proposed in [26] and modified by us in [27] (also see [28] for details of its application to dwarf stars of spectral class M, and [29]—to the superflare stars). An undoubted advantage of this technique consists in the possibility of its use given the sufficiently large samples of objects for the subsequent statistical analysis and establishing the dependences of a general nature.

The spottedness parameter S is defined as the ratio of the area of all the spots on the stellar surface to the area of its entire visible surface. Note that the technique proposed in [27] actually indicates only the S variation amplitude of the maximally spotted hemisphere of a star compared with the opposite one. Since we do not know the level of the star's brightness in the absence of spots on its surface, only the bottom limit of the spottedness value can be obtained. The explanations regarding the accuracy of determination of S , and among them for the solar-type stars can be found in [25].

In addition, we surveyed the results of analysis of young solar-type stars aged about 120–650 Myr (except for one rapidly-rotating star BD –07 2388) presented in [17]. These young objects have T_{eff} lower than the Sun and the stars we selected from [9] in Fig. 2a. The masses of stars studied are in the range of magnitudes of 0.9–0.95 solar masses, and the periods—in the range of 6–11 days.

The DR parameter in these objects is higher than that of similar stars from [9] (Figs. 2a and b). These objects are in a similar manner discernable in the ' $\Delta\Omega$ –Rossby number' diagram (Fig. 2c).

Unfortunately, for stars from [17] there are no area estimates of spots on their surface, hence Fig. 2d presents the data on the S parameter only for stars from [9]. Nevertheless, if we use this data alone, we can make a conclusion on the absence of a pronounced relationship between the S and Ro parameters. At the same time, for stars in a broad range of the S parameter variations, from 0.3% to 3% of

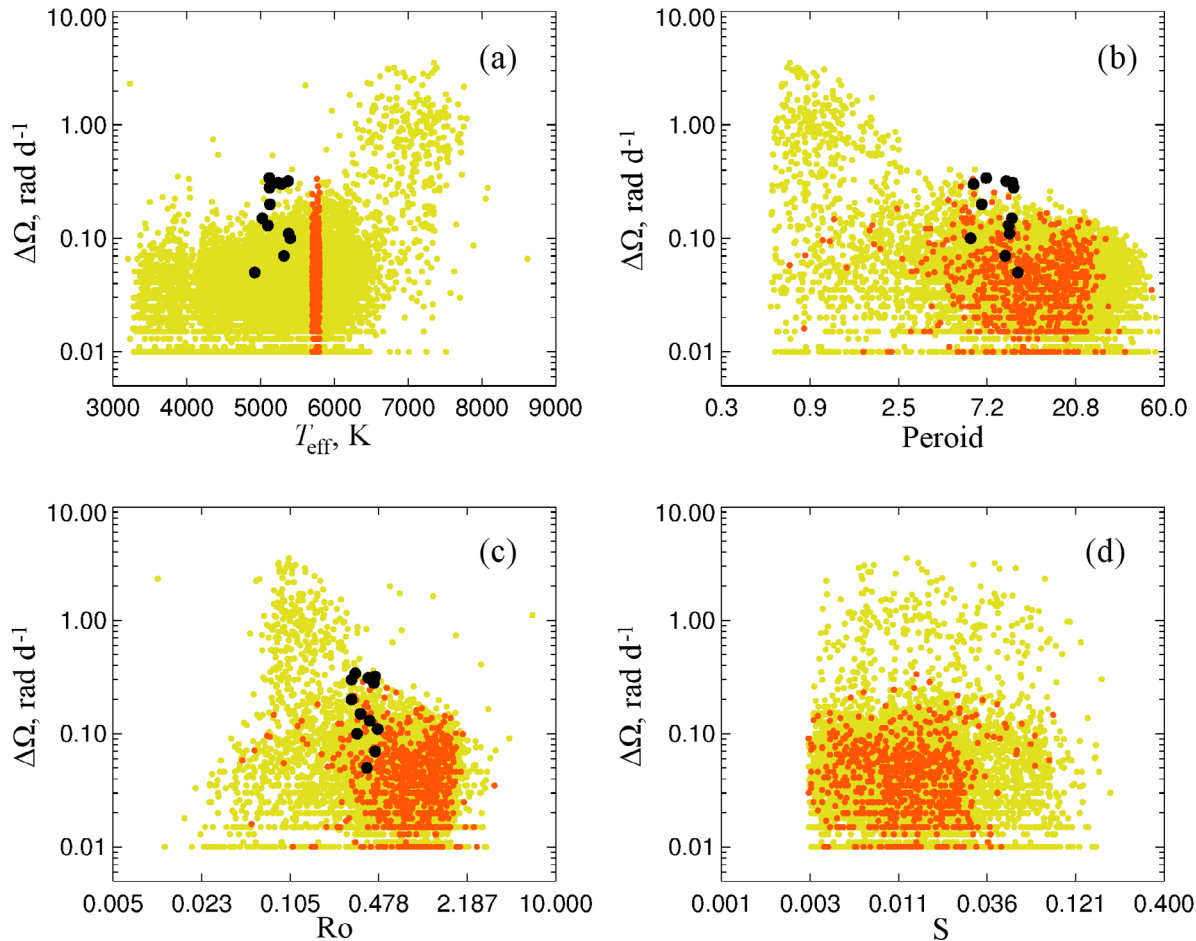


Fig. 2. Diagrams of variations of the $\Delta\Omega$ parameter with the temperature (a), period (b), Rossby number Ro (c) and the spottedness parameter S (d). The bright circles represent the stars from [9], the gray circles—solar-type stars from [9], and large dark circles—the data from [17].

the visible surface of the star, the $\Delta\Omega$ parameter also varies widely from 0.01 to 0.10.

Figure 3 shows the diagram of variations of the $\Delta\Omega$ parameter depending on age for the considered solar-type stars. The data on the age of objects is adopted from the studies [9] and [17]. According to our integrated data set, we can conclude that there are no noticeable variations in $\Delta\Omega$ with age. This is consistent with a similar conclusion of the authors of [17], as well as their conclusion about the absence of well-pronounced dependences between $\Delta\Omega$ and other parameters—the magnetic field strength, the periods of rotation and Rossby numbers. Based on this, the authors of [17] put the question of a weak effect of DR on the generation of magnetic fields in the stars they consider.

To some extent, we considered a similar situation in [29] within the study of superflare stars (with the energies of 10^{33} – 10^{36} erg), when we used all the results available at the time of their research. The

analysis of data from [29] allowed us, based on the independent determinations of brightness variability to confirm the conclusion on the increased spottedness of the surfaces of superflare stars. These same data [29] suggest that despite the increased spottedness, superflare stars do not stand out from the total star array with their DR parameters.

In [25] we performed a comparison of dependences of the chromospheric activity index R'_{HK} and the spottedness parameter S of the stars studied on age.

It was found that these dependences share common distinguishing characteristics indicating an increased level of activity for a part of young stars (younger than 3–4 billion years), relatively small activity variations of objects with age 4–8 billion years and a possible further decrease of activity in older stars. If we only consider the data from [17] and the $\Delta\Omega$ estimate for the Sun, then an assumption can be made on a similar variation of the $\Delta\Omega$ parameter

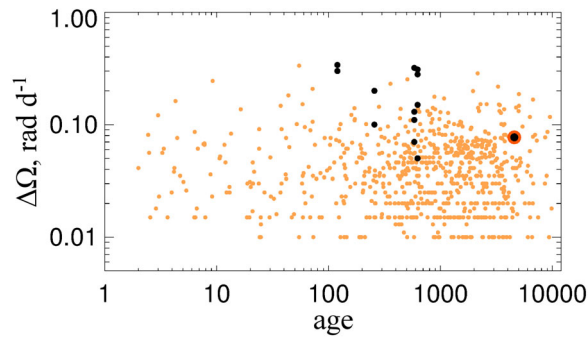


Fig. 3. A diagram of variations of the $\Delta\Omega$ parameter with age for the considered solar-type stars from [9] (bright circles) and [17] (large dark circles). The age of objects was adopted from [9] and [17]. A large contoured circle denotes the data for the Sun.

with age (its decrease). From the complete set of data from [9] such a conclusion can not be made.

5. ANTI-SOLAR DIFFERENTIAL ROTATION

The sign of the dimensionless α parameter in the ratio for the differential rotation law ($\Delta\Omega = \alpha \Omega_{\text{eq}}$, $\alpha_{\odot} = 0.2$) determines the type of DR, namely, it is solar with $\alpha > 0$, and anti-solar if $\alpha < 0$. Given the solar DR, the poles rotate slower than the equator, while at anti-solar DR they rotate faster. Most of the above techniques of finding the DR parameters give their absolute (without the sign) values. The data about the sign of α is given in the literature quite rarely. To date, the list of objects with negative α is small. According to [10], in a sample of 24 objects, 11 stars possess anti-solar DR. All of them are giants or subgiants, while three of them are of particular interest, these are single rapidly rotating giants with high lithium abundances, the nature of the occurrence of which remains unknown.

Theoretical studies of the causes of occurrence of anti-solar DR can be found in [30], a list of later publications—in [31].

According to the theoretical estimates [31], cool stars with large Rossby Ro numbers should be the objects with anti-solar DR. In Fig. 4, we present a diagram of dependence of the α parameter on Ro for objects from our integrated sample (for comparison, the data from [9] are also presented). Recall that our sample includes 11 objects with negative α from [10]. In a complete accordance with the prediction of [31] these objects lie on the diagram in the region of high Ro parameter values. A common course of variations in the α parameter depending on Ro for objects from our integrated sample is the same as for the stars from [9], the division of objects into groups with $\alpha < 0$ and $\alpha > 0$ occurs when Ro is about 0.1. Our diagram in Fig. 4 qualitatively resembles the diagram in Fig. 2

from [31], on which the separation of objects occurs with the entered in [31] value of Ro_c (the Rossby convective number) of about 1. The study [9] considers quite a number of objects with large Ro values, but since the method of analysis did not allow to determine the sign of the α parameter, it is unknown how many of them could fill the area of negative values we are interested in.

Reinhold and Arlt [32] made an attempt to develop a methodology for determining the sign of the DR α parameter based on the analysis of the first harmonics of a rotation period of a spotted star. A clarification of this technique is presented in the study [33], where the authors also considered the possibility of in some cases obtaining the data on the latitude of the spots on the stellar surface according to the data of photometric observations. In case of a successful development of the proposed methods the researchers would make significant progress in determining the sign of α , and the number of known stars with anti-solar DR would probably be significantly increased.

6. CONCLUSION

In the present survey we have discussed the methods of analyzing the observational manifestations of differential rotation (DR). Using the example of analyzing the ‘ $\Delta\Omega - T_{\text{eff}}$ ’, ‘ $\Delta\Omega - P$ ’ and ‘ $\Delta\Omega - Ro$ ’ diagrams, we made a comparison of determinations of DR stars with the results of theoretical research. It is shown that the empirical dependence, proposed by the authors of [5], characterizing the objects with $T_{\text{eff}} < 5000$ K can be used to represent the data of our larger sample in a wider temperature range. Particular attention is paid to the problems of measuring the DR parameters in low-mass dwarfs, including the fully convective ones, and their comparison with the theoretical data.

We analyzed the results of studies determining the DR parameters in solar-type stars.

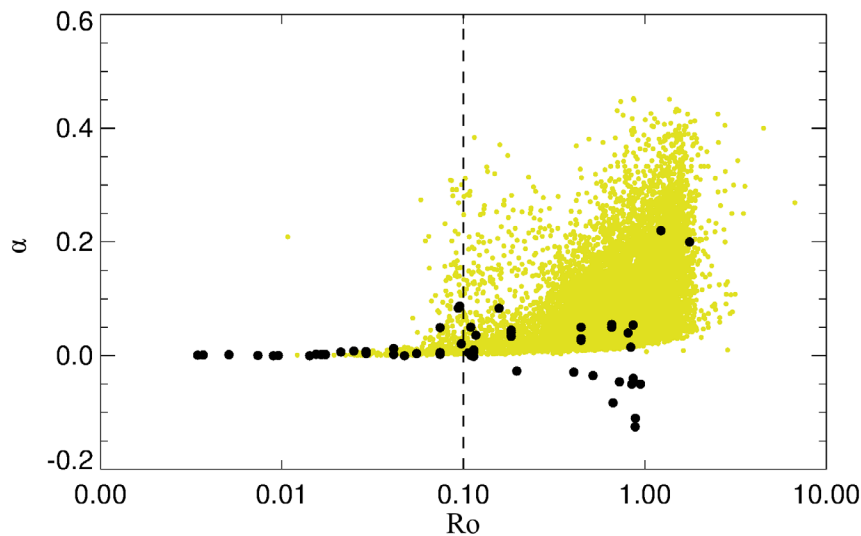


Fig. 4. The dependence of the α parameter on the Rossby number Ro for the objects of our integrated sample (the dark circles) (the data from [9] is also presented for a comparison—the small bright circles). Splitting the objects into groups with $\alpha < 0$ and $\alpha > 0$ occurs when Ro is about 0.1 (the vertical dashed line).

We considered the objects with anti-solar DR ($\alpha < 0$), and possible methods allowing to determine the sign of α , the use of which will lead to a significant increase in the number of known stars with anti-solar DR. We studied the properties of the dependence of α parameter on the Rossby number Ro .

In conclusion, let us note two areas of research that we believe would in the future expand our understanding of the DR manifestations.

We presented the survey of methods for studying the surface DR of stars at the beginning of the present paper. A more difficult task is the study of DR inside the stars. Only relatively recently the high-accuracy photometric data obtained by the space telescopes, and first of all, by the Kepler space telescope, allowed to solve this problem using the asteroseismology methods. There are by now only a few studies of this nature for red giants, cool subgiants and several rapidly rotating stars of spectral class F (see [34] and references therein). An example of use of the technique for determining the DR of five solar-type stars [34] has demonstrated its promising capabilities when used as the initial data for modeling the data on the magnitude of the surface DR.

The study [35] pointed to another important application of the DR research that the authors came across in the analysis of the eclipsing-variable systems. They related the presence among them of a group of subsynchronized systems with the manifestations of DR. Almost three quarters (77%) of eclipsing variable systems with a rotational brightness modulation caused by the spots reveal a splitting

of the peaks in the power spectra, possibly due to the presence of spots on various latitudes and DR. In such systems, the orbital rotation of the primary components with spots is probably synchronized with the period of proper rotation of the star at the equator, while the existence of spots in the polar regions leads to the observation of periods greater than the orbital ones.

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