Recent Trends in the Study of Compact Objects: Theory and Observation
ASI Conference Series, 2013, Vol. 8, pp 19–25
Edited by Santabrata Das, Anuj Nandi and Indranil Chattopadhyay

Viscous accretion disc around black holes with variable adiabatic index

Indranil Chattopadhyay[1*] and Rajiv Kumar[1,2]
[1]*ARIES, Manora Peak, Nainital 263002, Uttarakhand, India*
[2]*School of Studies in Physics & Astrophysics, Pt. Ravishankar Shukla University, Raipur - 492010, India*

Abstract. We study the viscous advective accretion disc around non-rotating black holes with variable adiabatic index. In this study we have developed all possible global accretion solutions e.g. Bondi type, smooth and shocked solutions with and with out viscosity with multispecies fluids and as well as standing shock parameter space with and without viscosity for different composition parameter.

Keywords : black hole physics – accretion, accretion discs – hydrodynamics

1. Introduction

Accretion onto black holes is the most favoured model to explain luminosities, variabilities and mass ejections from AGNs and microquasars. Therefore, understanding the physics of accretion is of paramount importance. Purely Bondi flow onto a black hole (Bondi 1952) is of low luminosity, and Keplerian disc produces high luminosity (Shakura & Sunyaev 1973) but cannot account for non-thermal radiation. The advective model and two component accretion flow model (TCAF) (Chakrabarti 1989; Chakrabarti & Titarchuk 1995; Chakrabarti 1999) can qualitatively explain the observations from black hole candidates in all frequency range. However, most of these works were done with fixed Γ (adiabatic index) equation of state (EoS). A black hole accretion from infinity to horizon cannot be described by a fixed Γ EoS, since for ultra-relativistic temperature (T) the flow is described by $\Gamma = 4/3$, for non-relativistic T it is $\Gamma = 5/3$, and generally it will be $5/3 > \Gamma > 4/3$. Inviscid accretion models with relativistic EoS and in general relativistic regime, were employed by various authors (Fukue 1987; Chattopadhyay 2008; Chattopadhyay & Ryu 2009; Chattopadhyay & Chakrabarti 2011; Kumar et al. 2013). The temperature range of the advective accretion flow is $10^6 K < T < 10^{11} K$, which warrants the fluid to be at least fully ionized. This means that such a flow if composed of similar kind of particles, then it should

*email: indra@aries.res.in

be electron-positron fluid ($e^- - e^+$), since a fluid composed of oppositely charged free protons or other heavier particles is a physical improbability. These authors have also shown that fluids composed of dissimilar particles for *e.g.,*electron-proton ($e^- - p^+$) show highly energetic phenomena like shock in accretion. However, Chattopadhyay (2008) showed that $e^- - e^+$ is much colder and much less relativistic than any fluid composed of dissimilar particles. In fact they showed that although $e^- - p^+$ is much hotter and thermally much more relativistic than $e^- - e^+$, but the most relativistic fluid is the one with proton proportion $\sim$ 20% of electron number density.

In this paper, we employ Newtonian equations of motion with relativistic EoS, and the properties of Schwarzschild metric will be mimicked by the pseudo-Newtonian (hereafter, PW) potential (Paczyński & Wiita 1980). We study viscous accretion flow with relativistic EoS around black holes and show that shocks do form in a significant range of boundary conditions. In the next section, we present the equations and in the last section we present the solutions and the discussion.

2. EoS and accretion model with multispecies fluids

We consider steady state, viscous, rotating and axis symmetric accretion disc around a Schwarzschild black hole. The hydrodynamic Navier-Stoke equation for an accretion disc in cylindrical co-ordinate (r, ϕ, z) is written as: The radial momentum equation

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{dp}{dr} - \frac{\lambda^2}{r^3} + \frac{1}{2(r-1)^2} = 0. \tag{1}$$

the angular momentum distribution equation

$$\frac{d\lambda}{dr} + \frac{1}{\Sigma v r}\frac{d(r^2 W_{r\phi})}{dr} = 0, \tag{2}$$

and the z component is assumed to be in hydrostatic equilibrium and gives the local disc half height $H = 2(r-1)\sqrt{\Theta r/\tilde{t}}$. The integrated continuity equation is written as

$$\dot{M} = 2\pi\Sigma v r, \tag{3}$$

where, $\dot{M}$ is the mass accretion rate of the flow and $\Sigma = 2\rho H$ is the vertically integrated density. Other local flow variables v, p, λ and ρ are fluid velocity, thermal pressure, specific angular momentum and density of the flow, respectively. $W_{r\phi} = \Sigma \nu r(d\Omega/dr)$ is $r\phi-$ component of viscous stress tensor and $\nu = \alpha a^2/(\Gamma\Omega_K)$, where a, Ω, and Ω_K are local sound speed, local angular velocity, and local Keplerian angular velocity, respectively. The unit of distance is $r_g = 2GM_{BH}/c^2 = 1$, unit of time $t_g = r_g/c = 1$ and unit of mass is the black hole mass $M_{BH} = 1$. The entropy generation equation,

$$\Sigma v\left(\frac{d\bar{e}}{dr} - \frac{p}{\rho^2}\frac{d\rho}{dr}\right) = Q^+ - Q^-, \tag{4}$$

where $\bar{e}$ is specific energy density. If e is the energy density then $\bar{e} = e/\rho$. Here, $Q^+ = W^2_{r\phi}/\eta$ and Q^- are heating and cooling term, but we consider only the heating term. The isotropic pressure (p) and mass density (ρ) are defined as

$$p = 2n_{e^-}kT \quad \text{and} \quad \rho = n_{e^-}m_{e^-}\tilde{t}, \tag{5}$$

where, $\tilde{t} = (2 - \xi + \xi/\eta)$, $\xi = n_{p^+}/n_{e^-}$ is ratio of proton number density to the electron number density, $\eta = m_{e^-}/m_{p^+}$ is ratio of electron mass to proton mass, k is Boltzmann constant and T is temperature of the flow. EoS for multispecies fluids which we have used is given by (Chattopadhyay 2008; Chattopadhyay & Ryu 2009),

$$e = n_{e^-}m_{e^-}c^2 f, \tag{6}$$

where,

$$f = (2-\xi)\left[1 + \Theta\left(\frac{9\Theta + 3}{3\Theta + 2}\right)\right] + \xi\left[\frac{1}{\eta} + \Theta\left(\frac{9\Theta + 3/\eta}{3\Theta + 2/\eta}\right)\right]. \tag{7}$$

Here, $\Theta = kT/(m_e c^2)$ is the dimensionless temperature of the flow. Hence $\bar{e} = c^2 f/\tilde{t}$. The enthalpy of the flow is defined as

$$h = (e + p)/\rho = (f + 2\Theta)/\tilde{t}. \tag{8}$$

Now integrating Eq.(1) with the help of Eqs.(2-4, 8), we get

$$E = \frac{v^2}{2} + h - \frac{\lambda^2}{2r^2} + \frac{\lambda\lambda_0}{r^2} - \frac{1}{2(r-1)}, \tag{9}$$

and is known as specific grand energy of the flow (Gu & Lu 2004). E is constant through out the flow even in presence of viscosity. Putting RHS of Eq. (4) as zero, and integrating it with the help of Eqs.(5-8), we obtain the adiabatic equation of state,

$$\rho = \mathcal{K}\exp(k_3)\,\Theta^{3/2}(3\Theta + 2)^{k_1}(3\Theta + 2/\eta)^{k_2}, \quad \text{where,}\mathcal{K} = \text{ adiabatic constant.} \tag{10}$$

Here, $k_1 = 3(2-\xi)/4$, $k_2 = 3\xi/4$, and $k_3 = (f - \tilde{t})/(2\Theta)$. Using equations (3) and (10), we can define entropy accretion rate ($\dot{\mathcal{M}}$) as

$$\dot{\mathcal{M}} = \frac{\dot{M}}{4\pi\mathcal{K}} = vH\exp(k_3)\,r\Theta^{3/2}(3\Theta + 2)^{k_1}(3\Theta + 2/\eta)^{k_2}. \tag{11}$$

$\dot{\mathcal{M}}$ is also a constant for inviscid multispecies relativistic flow. Simplifying Eq. (4) by using Eq. (3), we have

$$\frac{d\Theta}{dr} = -\frac{1}{(2N+1)}\left[\frac{2\Theta}{v}\frac{dv}{dr} + \frac{3\Theta}{r} + \frac{2\Theta}{r-1} + \frac{\tilde{t}v}{\nu r^2}(\lambda - \lambda_0)^2\right] \tag{12}$$

And Eq. (1) gives velocity gradient equation as

$$\frac{dv}{dr} = \frac{\frac{a^2}{\Gamma+1}\left(\frac{5r-3}{r(r-1)}\right) + \frac{v(\lambda-\lambda_0)^2}{(2N+1)\nu r^2} + \frac{\lambda^2}{r^3} - \frac{1}{2(r-1)^2}}{v - \frac{a^2}{v}\frac{2}{\Gamma+1}} = \frac{\mathcal{N}}{\mathcal{D}}. \tag{13}$$

Accretion onto a black hole is transonic, because supersonic inner boundary and subsonic outer boundary. Therefore, at certain distance $\mathcal{N} = 0$ and $\mathcal{D} = 0$, this gives the

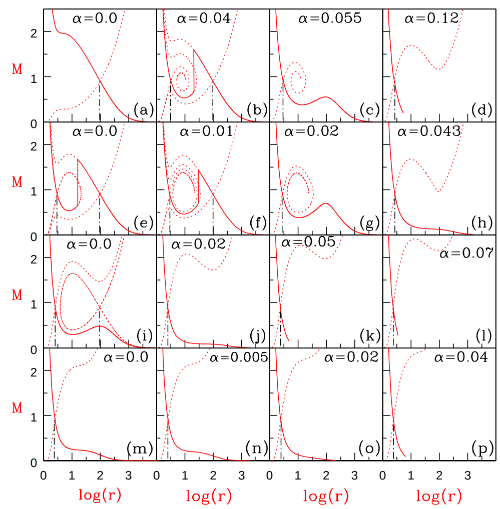

Figure 1. We plot Mach number ($M = v/a$) with $log(r)$ for different viscosity parameter (α). The solutions for parameters $(E, \lambda_0) = (1.001, 1.45)$ (a-d), $(E, \lambda_0) = (1.001, 1.55)$ (e-h), $(E, \lambda) = (1.001, 1.65)$ (i-l), and $(E, \lambda) = (1.004, 1.65)$ (m-p). Viscosity parameter α, increases towards right, and $\xi = 1.0$ for all the plots. The vertical dash-dotted line indicate the sonic point of the flow and solid lines are accretion solutions.

sonic point conditions.

Shock Conditions : The Rankine-Hugoniot shock conditions are obtained from conservation of mass flux $[\dot{M}] = 0$, momentum flux $[W + \Sigma v^2] = 0$ and energy flux $[E] = 0$. The shock front is infinitesimally thin, so we assume that $d\Omega/dr$ is continuous across the shock, the angular momentum jump condition is obtained by considering the conservation of λ flux across the shock. The jump conditions are,

$$v_- = \frac{c_0 + \sqrt{c_0^2 - 8\Theta_-/\tilde{t}}}{2}, \quad \lambda_- = \lambda_0 + \frac{c_2 a_-^2}{\Gamma_- v_-}, \text{ and, } f(\Theta) = \frac{v_-^2}{2} + h_- - \frac{\lambda_-^2}{2r_-^2} - \frac{\lambda_- \lambda_0}{r_-^2} - c_1. \tag{14}$$

Here, $c_0 = (2\Theta_+/\tilde{t} + v_+^2)/v_+$, $c_1 = v_+^2/2 + h_+ - \lambda_+^2/(2r_+^2) + \lambda_+\lambda_0/r_+^2$ and $c_2 = v_+\Gamma_+(\lambda_+ - \lambda_0)/a_+^2$. Subscripts + and − denote the post- and pre-shock variables, respectively.

3. Solutions and discussions

Due to the coordinate singularity on the horizon, we have to estimate the flow variables close to the horizon, which is $r_{in} = 1.1$. Since after integration Eq. (2) remains a first order differential, so we can expand it by Frobenius expansion for $\lambda(r)$ about $r = 1$ (Becker & Le 2003).

$$\lambda(r) = \lambda_0 + B(r-1)^\beta, \qquad r \to 1 \tag{15}$$

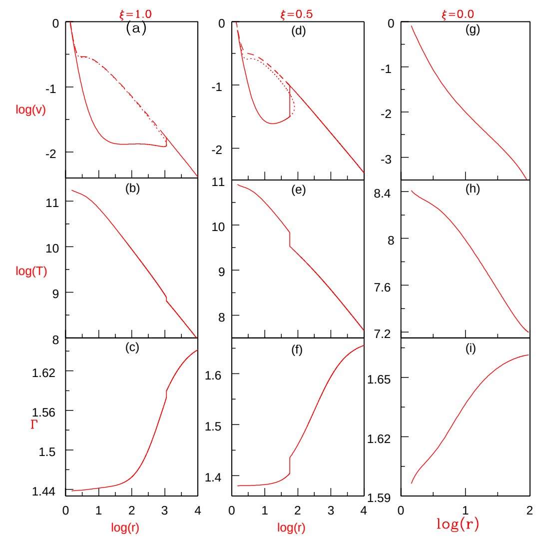

Figure 2. We plot $log(v)$ (a, d, g), $log(T)$ (b, e, h), and Γ (c, f, i) with $log(r)$ for $\xi = 1.0$ (a-b), $\xi = 0.5$ (d-f) and $\xi = 0.0$ (g-i). The flow parameters of these plots are $E = 1.000001, \alpha = 0.01, \lambda_0 = 1.7$. For $\xi(= 1.0, 0.5)$, we have shocks and they occur at $r_s = 1110.82, r_s = 57.03$, respectively. But $\xi = 0.0$ gives smooth solution and only one sonic point at $r_{ci} = 2.238$.

where, $B = 4\sqrt{2\Theta}\lambda_0 \alpha a^2 r_g \rho_e/(\Gamma\sqrt{\tilde{t}}\dot{\mathcal{M}}), \beta = 2$ and $\rho_e = e^{k_3}\Theta^{3/2}(3\Theta+2)^{k_1}(3\Theta+2/\eta)^{k_2}$. By providing the flow parameters (E, λ_0, α and $\dot{\mathcal{M}}$), we obtain $\lambda(r_{in})$ from Eq. (15). Now we can integrate Eqs. (2, 12 and 13) outward from r_{in} with the help of Eqs. (9 and 11). The correct solution through the critical point is found by supplying the appropriate $\dot{\mathcal{M}}$.

In Fig. 1a-p, we have plotted the Mach number M $(= v/a)$ with $log(r)$. The solutions are for $(E, \lambda_0) = (1.001, 1.45)$ (a-d), $(E, \lambda_0) = (1.001, 1.55)$ (e-h), $(E, \lambda) = (1.001, 1.65)$ (i-l), and $(E, \lambda) = (1.004, 1.65)$ (m-p). The viscosity parameter α (marked on the panels), increases left to right. All the plots are for $\xi = 1.0$. In case of low λ_0 flow (Fig. 1a-d), the inviscid solution is Bondi type, characterized by a single sonic point r_{co} far away from the horizon. Keeping the same E & λ_0 but increasing α tantamount to higher λ away from the horizon, which may support accretion shocks. In this particular case the lower limit of α which triggers shock formation is $\alpha_l = 0.03147$. As α is increased, λ increases in the outer part of the disc. So more and more energy is going to the rotational head rather than the radial kinetic energy head, and hence the flow cannot become supersonic at reasonable distance away from the horizon, and consequently shocks may not form. The upper limit of viscosity parameter to form shock for these flow parameters is $\alpha_u = 0.0494$. For parameters $(E, \lambda_0) = (1.001, 1.55)$ (Fig. 1e-h), shock forms even for $\alpha = 0.0$, but beyond $\alpha_u = 0.01469$ no shock is found to form. For higher values of flow parameters, shock transitions are not found (Fig. 1i-p). In Fig.2a-i, we fix $(E, \lambda_0$ & $\alpha) = 1.000001, 1.7, 0.01$, and vary ξ and compare the solutions. In Fig. 2a-c, $\xi = 1.0$, in Fig.2d-f $\xi = 0.5$, and in Fig.2g-i

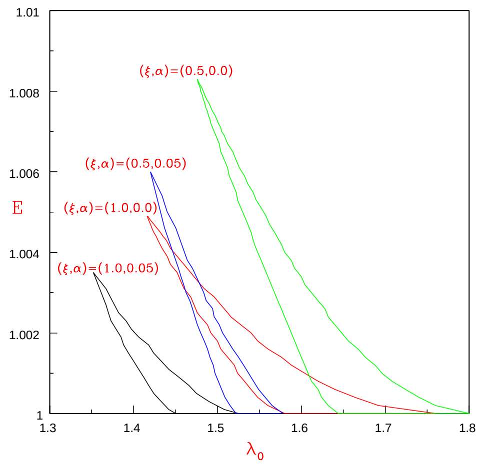

Figure 3. Represents $E-\lambda_0$ shock parameter space with and with out viscosity for $\xi, = 1.0, 0.5$. The α values are mentioned on the figure.

$\xi = 0.0$. The flow variables plotted are $log(v)$ (a, d, g), $log(T)$ (b, e, h), and Γ (c, f, i). While $\xi \neq 0.0$ shows shock transition, but $\xi = 0.0$ is a smooth solution with an inner sonic point $r_{ci} = 2.238$. Although models with $\xi \neq 0.0$, do show shock but the locations are at large distance apart. Moreover, the solutions themselves are different too. This implies accretion solutions crucially depend on its composition, and the emergent spectra will be different too. In Fig.3, we plot the shock domain in the $E - \lambda_0$ parameter space, for $(\xi,\ \alpha) = (1.0, 0.0)$, $(\xi,\ \alpha) = (1.0, 0.05)$, $(\xi,\ \alpha) = (0.5, 0.0)$, and $(\xi,\ \alpha) = (0.5, 0.05)$. For a given α, the right ward shift of the shock domain with decreasing ξ, which has also been shown for inviscid disc (Chattopadhyay 2008; Chattopadhyay & Chakrabarti 2011; Chattopadhyay et al. 2012). While increasing α for a given ξ, the parameter space shifts to the left, a typical behavior of viscous flow (Chakrabarti & Das 2004; Kumar & Chattopadhyay 2013; Kumar et al. 2013). Therefore the overlapping region changes with either the change of α or ξ.

In this paper, we have shown that all type accretion solutions are possible for viscous flow with realistic EoS. In fact one may identify, amongst other solutions, the ADAF type solutions (Narayan et. al. 1997) in Fig. 1h, j &n, and which are a part of the general advective solutions. We have shown that, viscous disc solutions with a variable Γ EoS show shocks, similar to the inviscid solutions, however, only EoS containing both electrons (positrons) and protons exhibit shock, while EoS of pair plasma do not show shock. We know to produce accretion shocks, the flow should be hot (semi-relativistic T) at least for $r \lesssim 1000$, but pair plasma barely reaches $T \sim 10^8$K close to the horizon, so the effective boundary layer is not possible for pair plasma, therefore shocks are not formed for $e^- - e^+$. The most interesting solution would be the one where particle creation processes are considered, so that ξ becomes variable and not a global constant.

References

Bondi H., 1952, MNRAS, 112, 195
Becker P. A., Le T., 2003, ApJ, 588, 408
Chakrabarti S. K., 1989, ApJ, 347, 365
Chakrabarti S K., Titarchuk L., 1995, ApJ, 455, 623.
Chakrabarti S. K., 1999, A&A, 351, 185
Chakrabarti S. K., Das S., 2004, MNRAS, 349, 649
Chattopadhyay I., 2008, AIP Conf. Ser. Vol. 1053, Am. Inst. Phys., New York, p. 353
Chattopadhyay I., Ryu D., 2009, ApJ, 694, 492
Chattopadhyay I., Chakrabarti S. K., 2011, Int. Journ. Mod. Phys. D, 20, 1597
Chattopadhyay I., et al., 2012, BASI, ASI Conf. Ser. Vol. 5, pp 81-89
Fukue J., 1987, PASJ, 39, 309
Gu W. M., Lu J. F., 2004, ChPhL, 21, 2551
Kumar R., Singh C. B., Chattopadhyay I., Chakrabarti S. K., 2013, MNRAS, doi 10.1093/mnras/stt1781
Kumar R., Chattopadhyay I., 2013, MNRAS, 430, 386
Narayan R., Kato S., Honma F., 1997, ApJ, 476, 49
Paczyński B., Wiita P. J., 1980, A&A, 88, 23
Ryu D., Chattopadhyay I., Choi E., 2006, ApJS, 166, 410
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337