Modeling of Deformable MEMS Mirrors in Modern Telescopes

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Abstract: Ground based modern telescopes with AO (Adaptive Optics) are at par with the space borne telescopes in providing unprecedented quality images. Recently, deformable mirrors using MEMS (Micro-Electro-Mechanical Systems) have become a popular choice for adaptive mirrors due to various advantages. The continuous facesheet of the MEMS mirror can be modeled with the help of theory developed for thin plates [1], [2] and [3]. In this paper we discuss the modeling techniques using energy principles and variational methods [4] and [5]. For modeling and simulations we will follow the specifications of a commercially available 144 actuator continuous facesheet deformable MEMS mirror by Boston Micromachines Corporation [6]. The dynamics of this mirror is very fast and hence is neglected when compared to the rate of corrections to be applied and it is assumed that the boundaries of the mirror are simply supported. Thus our problem simplifies to that of a simply supported thin plates static under equilibrium condition. The MEMS mirror equation under the influence of point load matrix is obtained using superposition principle and Navier solution method is used for solving the deformation matrix for a given force matrix. In the case of an AO system, first the atmospheric wavefront is measured which then gives the desired shape of the mirror. Hence the deformation matrix is known and it is required to derive the force matrix, which essentially means solving the inverse problem. If the measured wavefront has noise which is normally the case, or the transformation matrix is rank deficient, the inverse problem becomes ill-posed [7] and [8].

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1. MEMS Mirror Modeling and Solution

The generalized governing equation for thin plate in $x-y$ plane is derived using Hamilton's principle. For thin plates it is assumed that the thickness remains constant and hence elasticity equations for x and y dimensions are considered. The Lagrangian is given by $L = U + V - K$ where U is the total strain energy, V is the total potential energy and K is the total kinetic energy. Using Hamilton's principle the virtual Lagrangian is written as [5]

$$0 = \int_{t_1}^{t_2} \delta L dt = \int_{t_1}^{t_2} (\delta U + \delta V - \delta K)\, dt \tag{1}$$

In two dimensional case the the virtual energies are given as

$$\begin{aligned}
\delta U &= -\int_{\Omega_0} \left(M_{xx}\frac{\partial^2 \delta w_0}{\partial x^2} + M_{yy}\frac{\partial^2 \delta w_0}{\partial y^2} + 2M_{xy}\frac{\partial^2 \delta w_0}{\partial x \partial y} \right) dxdy \\
\delta K &= \int_{\Omega_0} \left[I_0 \dot{w}_0 \delta \dot{w}_0 + I_2 \left(\frac{\partial \dot{w}_0}{\partial x}\frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y}\frac{\partial \delta \dot{w}_0}{\partial y} \right) \right] dxdy \\
\delta V &= -\int_{\Omega_0} q(x,y)\delta w_0 dxdy
\end{aligned} \tag{2}$$

where w_0 is the deflection and M_{xx}, M_{yy} and M_{xy} are the moments per unit length, (I_0, I_2) are the mass moments of inertia, $q(x,y)$ is the distributed vertical force, Ω_0 represents the mid-plane surface. On using fundamental lemma of calculus of variation the following Euler-Lagrange equation can be obtained [5]

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$$D\left(\frac{\partial^4 w_0}{\partial x^4}+\frac{\partial^4 w_0}{\partial y^4}+2\frac{\partial^4 M_{xy}}{\partial x^2\partial y^2}\right)=q(x,y)-I_0\ddot{w}_0+I_2\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2}+\frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \tag{3}$$

Here the moments are replaced using moment-deflection relationship and D represents flexural rigidity . By setting the dynamics to zero, following plate equation for static case is obtained:

$$D\left(\frac{\partial^4 w_0}{\partial x^4}+\frac{\partial^4 w_0}{\partial y^4}+2\frac{\partial^4 M_{xy}}{\partial x^2\partial y^2}\right)=q(x,y) \tag{4}$$

For simply supported boundary conditions Navier's method [5] can be used to solve the linear plate equation 4. The solution for deflection of thin-plate to a vertical point load Q_0 at (x_0,y_0) is given by

$$w_0(x,y)=\frac{4Q_0}{ab\pi^4 D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{sin\frac{m\pi x_0}{a}sin\frac{n\pi y_0}{b}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\right\}sin\frac{m\pi x}{a}sin\frac{n\pi y}{b} \tag{5}$$

where a and b are dimensions of plate along x and y directions respectively. For 144 actuators arranged in a square along x and y directions with a pitch of $a/12$, where $a=b$, the net deformation at any (x,y) point can be determined using superposition principle. The net deformation will be a summation of deformations corresponding to the individual forces. Let the point force matrix be given by

$$\begin{bmatrix} Q_{1,1} & Q_{1,2} & \dots & Q_{1,12} \\ Q_{2,1} & Q_{2,2} & \dots & Q_{2,12} \\ \vdots & \vdots & \dots & \vdots \\ Q_{12,1} & Q_{12,2} & \dots & Q_{12,12} \end{bmatrix} \tag{6}$$

Let the plate corners be at $(0,0)$, $(12,0)$, $(0,12)$ and $(12,12)$, then $Q_{1,1}$ is the force applied at $(a/24,a/24)$, $Q_{1,2}$ is the force applied at $(3a/24,a/24)$, $Q_{2,1}$ is the force applied at $(a/24,3a/24)$ and so on with $Q_{12,12}$ being the force applied at $(23a/24,23a/24)$. Now the deformation will be given by

$$\begin{aligned} w(x,y)=&\frac{4Q_{1,1}}{ab\pi^4 D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{sin\frac{m\pi}{24}sin\frac{n\pi}{24}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\right\}sin\frac{m\pi x}{a}sin\frac{n\pi y}{b} \\ &+\frac{4Q_{1,2}}{ab\pi^4 D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{sin\frac{m\pi}{24}sin\frac{3n\pi}{24}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\right\}sin\frac{m\pi x}{a}sin\frac{n\pi y}{b} \\ &\dots \\ &+\frac{4Q_{12,12}}{ab\pi^4 D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{sin\frac{23m\pi}{24}sin\frac{23n\pi}{24}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\right\}sin\frac{m\pi x}{a}sin\frac{n\pi y}{b} \end{aligned} \tag{7}$$

For the MEMS mirror, when all 144 point forces are acting together the solution is obtained using the superposition principle as in Figure 1. In general the force-deformation mapping can be written as

$$\begin{bmatrix} w(1,1) & w(1,2) & \dots & w(1,12) \\ w(2,1) & w(2,2) & \dots & w(2,12) \\ \vdots & & & \\ w(12,1) & w(12,2) & \dots & w(12,12) \end{bmatrix} = G \begin{bmatrix} Q_{1,1} & Q_{2,1} & \dots & Q_{12,1} \\ Q_{1,2} & Q_{2,2} & \dots & Q_{12,2} \\ \vdots & & & \\ Q_{1,12} & Q_{2,2} & \dots & Q_{12,12} \end{bmatrix} \tag{8}$$

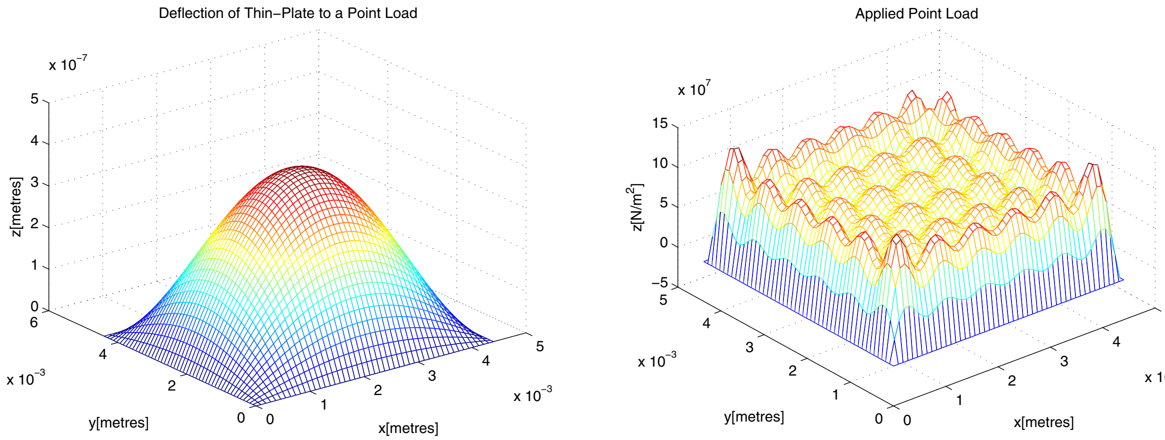

Fig. 1. Left: MEMS mirror deformation; Right: Applied 144 point loads

In the case of AO system we are interested in solving the inverse problem. The transformation G may or may not be invertible and also noise in measurements can render the inverse problem ill-posed. The SVD (singular value decomposition) technique gives a pseudoinverse where as regularization can be used for obtaining the optimal number of singular values. In the regularization method, solutions are selected to sacrifice model fit to data in exchange for solution stability.

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