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Identification of Friction in the 50/80 cm ARIES Schmidt Telescope Using the LuGre Model

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Abstract: In this paper, we discuss the LuGre model based parameter estimation of the friction present in the 50/80 cm Schmidt telescope at the Aryabhatta Research Institute of Observational sciencES (ARIES). Identification and compensation of friction in telescopes is essential to achieve sub-arcsec pointing and tracking accuracies at very low velocities. The six parameter LuGre model which is an extension of the Dahl model captures most of the observed dynamic behavior of friction. High accuracy encoders mounted on the telescope axes are capable of measuring micro and macro displacements useful for estimating the six parameters. A suitable PI controller was developed for performing experiments on the Schmidt telescope to estimate the friction. It was observed that the system exhibits the behaviour described by the LuGre friction model.

Keywords: LuGre model; Dahl model; Schmidt Telescope

1. INTRODUCTION

Friction is present in every mechanical system and creates many control related problems like large steady state errors and oscillations. At very low speeds, friction exhibits complicated dynamics and dominates the overall dynamics of the system. To achieve accurate control at low speeds, friction dynamics has to be modeled and compensated. Many friction models along with the compensation techniques are presented in Armstrong et al. [1994]. The LuGre model (Canudas et al. [1995]) which is an extension to the Dahl model (Dahl [1968]) effectively captures most of the observed behavior of dynamic friction and hence is widely accepted. This model has been used in Rivetta et al. [1998], Freidovich et al. [2010], Hensen et al. [2002] and Zhang et at. [2002] for friction identification and different compensation techniques have also been proposed in Armstrong [1991], Canudas et al. [1995] and Freidovich et al. [2010].

In this paper, we discuss the experimental determination of friction present in the 50/80 cm Schmidt telescope at Aryabhatta Research Institute of Observational sciencES (ARIES), Nainital, India, using the LuGre model for friction. Estimation of friction will aid in accurate modeling, analysis and development of suitable controller for achiev-ing desired performance. The paper is organized as follows: we discuss different aspects of the 50/80 cm Schmidt telescope in section 2, experimental setup and friction estimation using LuGre model is described in section 3 and concluding remarks of the paper are presented in section 4.

⋆ Author T. S. Kumar is on leave from Aryabhatta Research Institute of observational sciencES.

2. 50/80 CM SCHMIDT TELESCOPE

2.1 Telescope Objective

The 50/80 *cm* Schmidt telescope is an upcoming facility for astronomical observations in the night-time sky. The latitude and longitude of the site are 29 degrees 22 minutes and 79 degrees 27 minutes respectively. The telescope has a f/1 spherical primary mirror of 80 *cm* diameter with a usable aperture of 50 *cm* diameter. The telescope optics produces a 4 *deg* x 4 *deg* FOV at the prime focus. A large format KAF-16803 4096×4096, $9\mu m \times 9\mu m$ pixels Fingerlakes Proline CCD camera system is mounted on the focal plane to generate the sky data. Each pixel samples 3.5 *arcsec* x 3.5 *arcsec* of the sky.

Due to the rotation of the Earth, the position of the sky keeps drifting at a rate of 15 *arcsec/sec*, hence the objective of the controller is to accurately track the sky. Typically, a CCD exposure time of less than 20 minutes is required for generating useful sky data. The tracking speed of the telescope has to be accurate within an error of 1 *arcsec/sec* during the exposure. The controller has to be capable of positioning the telescope at the desired locations in the sky accurately with errors less than 10 *arcsec*. The desired speed range of telescope is between 1 *arcsec/sec* (4.85×10^{-06} *radians/sec*) to 2 *deg/sec* (3.49×10^{-02} *radians/sec*).

2.2 Telescope Structure

The telescope (Fig. 1) is supported on an equatorial mount. The polar axis is aligned parallel to the axis of rotation of the Earth and hence gets inclined to an angle defined by the latitude of the site. There are two motions in

the system: Right ascension (R.A.) motion along the polar axis and declination motion along an axis perpendicular to the polar axis. The off-axis tube is balanced by dead weights and gear box mounted on the other side.
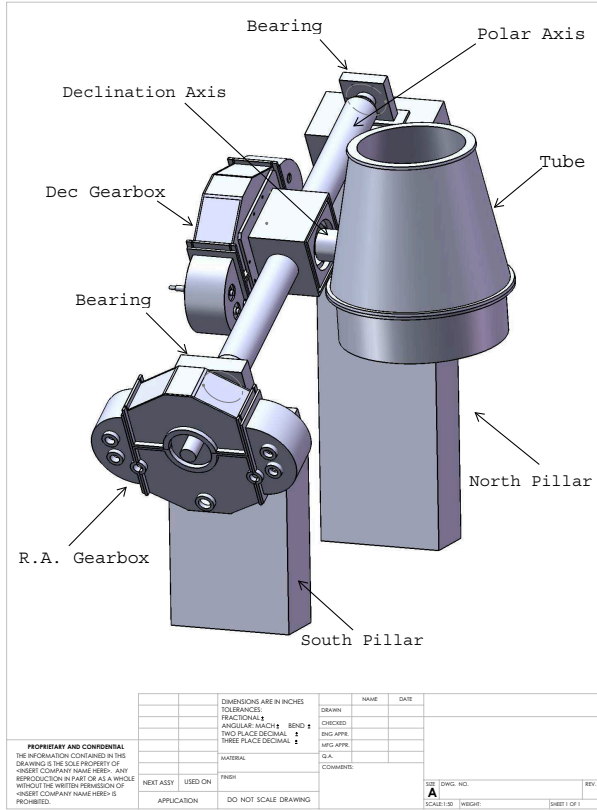

Fig. 1. Schematic of 50/80 cm ARIES Schmidt Telescope

The R.A. gear (Fig. 2) box housing is anchored to the south pillar with the help of stiff torque arms and provides an overall reduction of 360.489:1. To avoid backlash in the gears each axis is driven by a set of two identical motors coupled to the bull gear of polar axis through two identical gear trains. The two motors provide unequal torque to the bull gear such that if one motor leads the motion in forward direction the other leads the motion in the reverse direction. This technique ensures that the teeth contact is never lost between any of the gears during motion reversal and thus a dead zone is avoided. Coupling between the motor and the gear box is rigid as a direct drive DC motor is used. A friction roller assembly is available on the gear box for mounting an incremental encoder. This arrangement allows backlash free accurate velocity measurements through a reduction of 1:24.5. The drive system is identical for both R.A. and declination axis.

Assuming that the system is balanced, the generalized model for one axis of the telescope is given by

$$\tau(t) = J\ddot{\theta}(t) + T_{friction} \qquad (1)$$

where τ is the applied torque.

The telescopes are operated at low speeds and the controller bandwidth is kept below its fundamental mode, hence the torsional stiffness is usually ignored.
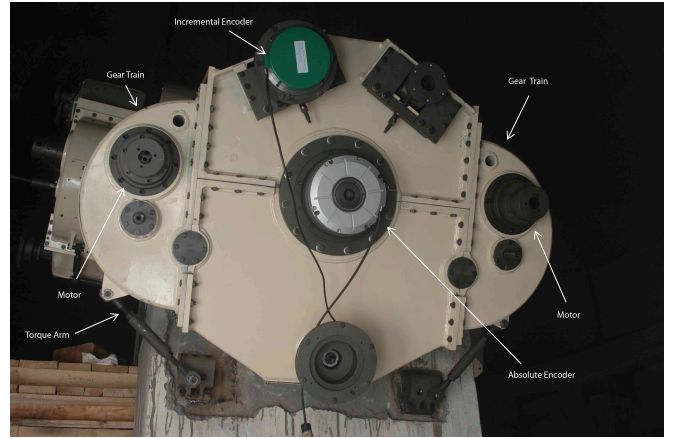

Fig. 2. R.A. Gear Box of the Schmidt Telescope

3. FRICTION PARAMETERS OF SCHMIDT TELESCOPE

Telescopes operate at very low velocities and are designed to provide high resolution images with sub-arcsec accuracies. For improving the performance of a telescope the system model has to be accurate and friction effects have to be compensated. A successful attempt was made by Rivetta et al. [1998] in identifying the friction parameters in the 2.5 m SDSS telescope using the LuGre model for friction.

3.1 The LuGre Model

The LuGre model characterized by a set of six parameters can be identified for a system by performing two set of experiments. The Stribeck effect is described by parameters F_C, F_S, σ_2 and v_s, which can be determined by measuring force at different steady state velocities. The parameters σ_0 and σ_1 describe the dynamics of friction and are determined by measuring presliding displacements.

The LuGre model given by Canudas et al. [1995] captures most of the friction dynamics in the four regimes of lubrication (Armstrong [1991]). The model is not proposed based on hydrodynamics of lubrication but is based on experimental observations and modeled to capture most of these observed behavior. It describes friction using the bristle model where the bristle dynamics is described by a nonlinear differential equation (2)

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)}.z \qquad (2)$$

where v is the relative velocity between the two surfaces and z is the bristle displacement or the presliding displacement.

The term $g(v)$ is a positive function and is parameterized as below (equation (3)) to capture the Stribeck effect (Armstrong [1991]).

$$\sigma_0.g(v) = F_C + (F_S - F_C)e^{-(v/v_s)^\delta} \qquad (3)$$

where F_C is the Coulomb friction, F_S is the stiction force, v_s is the Stribeck velocity and δ is an empirical parameter which can be experimentally determined. The value $\delta = 1$ is proposed by Tustin [1947], Bo et at. [1982] found δ to

range from 0.5 to 1 whereas Armstrong [1991] and Canudas et al. [1995] used $\delta = 2$.

The friction force which accounts for presliding displacement z, Stribeck effect and viscous friction is given as

$$F = \sigma_0.z + \sigma_1.\frac{dz}{dt} + \sigma_2.v \qquad (4)$$

where σ_0 is the stiffness of the bristles, σ_1 a damping coefficient during micro-sliding and σ_2 is the viscous coefficient.

Under steady state velocities, the bristle deflection will be constant z_{ss} and $\frac{dz}{dt}$ becomes zero. Using equation equation (2), z_{ss} can be written as

$$z_{ss} = g(v).sgn(v) \qquad (5)$$

The steady state friction as shown in equation (6) is a function of relative velocity between the surfaces and is obtained by substituting z_{ss} and $g(v)$ in equation (4)

$$F_{fss}(v) = \left[F_C + (F_S - F_C).e^{-(v/v_s)^\delta} \right].sgn(v) + \sigma_2.v \quad (6)$$

In case of a system undergoing rotational motion the equation (2) and equation (6) can be rewritten in terms of friction torque, angular displacements and angular velocities as

$$\frac{d\theta_z}{dt} = \omega - \frac{|\omega|}{g(\omega)}.\theta_z \qquad (7)$$

$$T_{fss}(v) = \left[T_C + (T_S - T_C).e^{-(\omega/v_s)^\delta} \right].sgn(\omega) + \sigma_2.\omega \quad (8)$$

The objective of the experiments in the next section is to estimate the friction in the Schmidt telescope using the LuGre model described by equation (7) and (4) which is characterized by a set of six parameters: $T_C, T_S, \sigma_0, \sigma_1, \sigma_2$ and v_s.

3.2 Experimental Setup

The telescope is assembled and dummy mirrors are mounted in place of the actual optical elements to prevent any damage. For performing the experiments the telescope was first properly balanced by monitoring the motor currents in open loop and adjusting the counter weights. Each axis is driven by two hollow shaft QT-3124 Kollmorgen brushed permanent magnet torque DC motors. These motors are available in frameless pancake configuration, i.e., large diameter and a narrow width, and assembled directly on the gear shaft. The motors are capable of operating continuously at stalling current. For velocity feedback, 4.608 million pulses per revolution Gurley, series 8×60, incremental encoder is used. Through reduction it provides an overall resolution of 0.0011 *arcsec* accurate upto 0.05 *arcsec*. A 29 bit Heidenhein hollow shaft absolute encoder provides direct measurement of the output shaft position to an accuracy of 0.5 *arcsec*.

A suitable hardware was developed for performing the experiment which mainly provides three functions: interfacing the encoders, providing control effort to the motors and generating useful data for logging and analysis. For maintaining a constant speed, an incremental PI controller in PWM mode was developed on one circuit board with the help of a dsPIC30f3011 microcontroller. Variation in

current reflects the friction hence it is used in feedback and only the velocity loop is closed. The encoder feedback module of dsPIC is used for capturing encoder pulses from the incremental encoder for estimating the velocity. The interrupt routine is used to ensure a fixed sampling time of 5 ms.

To drive the two motors, two IRF250 power MOSFETs along with the driver stage are used to amplify the PWM signals coming from the microcontrollers. To reduce backlash in the gear box the controller always applies 100% control effort on the forward direction motor and 10% on the reverse direction motor. The motor is connected between a 24 Vdc supply and the drain terminal of MOSFET and a 0.1 *ohm* resistance is connected between the source terminal of MOSFET and ground. An ADC channel of the dsPIC is used for sampling the voltage across the resistance to determine the motor armature current. The ADC module is synchronized with the PWM module using interrupt to capture the samples during the PWM falling edge where the motor current peaks.

Another board was developed using PIC18f4480 microcontroller to interface the position encoder using the synchronous serial port. The encoder provides serial data packets on the bidirectional EnDat interface which is decoded by the microcontroller and transmitted to the PC. These two boards consist of other peripherals for serial debugging, programming and asynchronous serial communication with the PC. Time stamped data packets each containing 25 sets of 10 ms samples of input signal, control effort, telescope position, telescope velocity, and ADC counts are continuously transferred to an interfacing PC at a fixed interval of 250 ms on RS232 serial port. The response of the controller at low and high velocities for both the directions is shown in Fig. 3. At 1.11×10^{-04} *radians/sec* the telescope operates in the Stribeck region and hence exhibits limit cycles due to a negative slope in the friction velocity curve resulting in large tracking errors. As the speed is increased to 3.90×10^{-02} *radians/sec*, the friction becomes a linear function of velocity and the controller is able to track the reference with errors less than 1%.
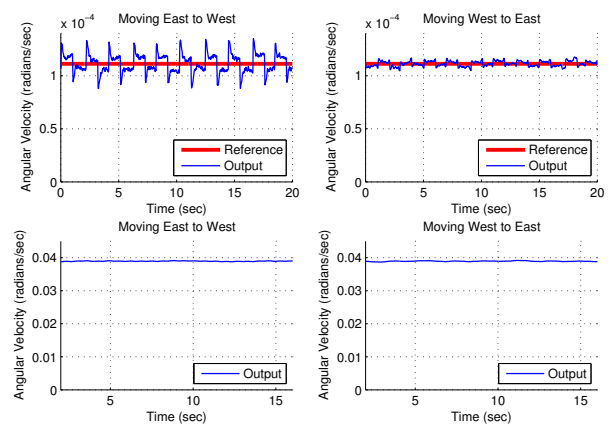

Fig. 3. Top: response at low speed; Bottom: response at higher speed

3.3 Estimation of Static Friction Parameters

The experiments done for the R.A. axis alone are discussed here as it is used for accurately tracking the sky. During the experiment, the declination axis is kept at a constant position. The R.A. axis is rotated at 30 different speeds in the range 1.11×10^{-05} to 4.45×10^{-02} *radians/sec* with the help of the PI controller.

At each speed the axis is moved 4 times from East to West and back, between the same initial and final positions. The experiments were repeated around the 0 degree zenith angle, 30 degree east, 60 degree east, 30 degree west and 60 degree west which covers the complete operating range of the R.A. axis of the telescope. The final data for each speed is generated by taking the average of the multiple set of readings taken at different positions. To reduce the effect of dwell time on friction the telescope is warmed up by moving it back and forth after completion of each run. A GUI was developed in Visual C++ for providing the command sequence for moving the telescope. It continuously displays the speed, position and current data and stores it on the hard disk in ASCII format.

At steady state velocities, $\frac{d\theta_z}{dt}$ (equation (7)) will be zero and friction is given by equation (8). Since acceleration is not present, friction can be measured by directly measuring the driving torque or motor armature current. Velocity is measured using the encoder mounted on the telescope. The friction at zero velocity corresponds to the breakaway torque required to just initiate the motion, when the system is subjected to a slow ramp input (Fig. 4).
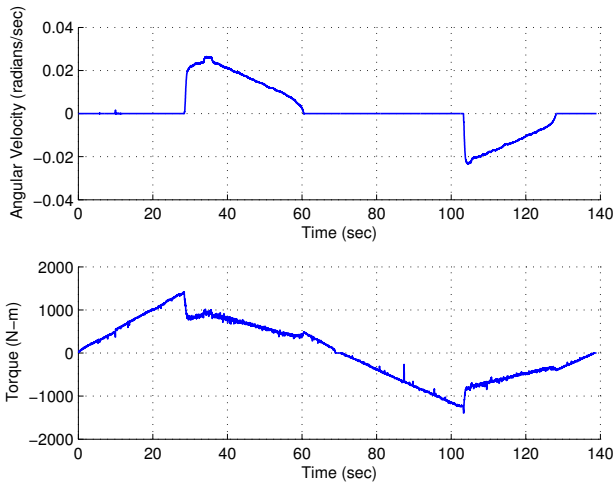

Fig. 4. Top: the system response; Bottom: the ramp input

The data analysis procedure is similar to that described in Rivetta et al. [1998]. Nonlinear optimization was performed in Matlab to fit equation (8) in the torque-velocity data by minimizing cost function:

$$\min_{\sigma_2, v_s, T_S, T_C} \sum_{i=1}^{n} \left[T_{fss}(\omega_i) - \hat{T}_{fss}(\omega_i) \right]^2 \qquad (9)$$

The estimated static parameters of friction for the LuGre model are shown in Fig. 5. While performing the nonlinear fitting, we found that $\delta = 0.85$ provides the best fit. The

friction characteristics need not be symmetrical for the two directions of motion.

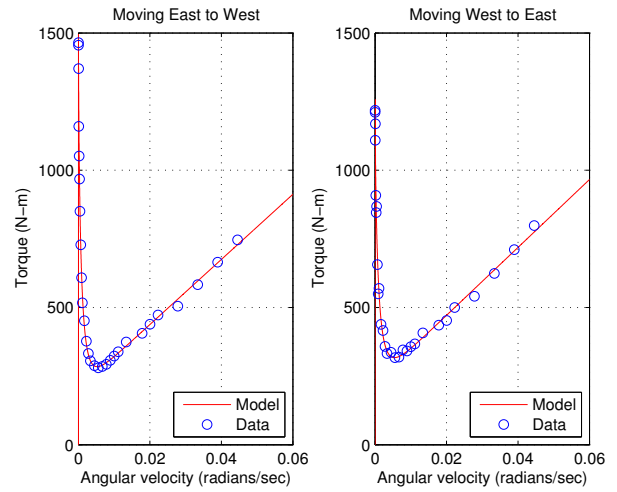Table 1 shows the estimated parameters which describe the Stribeck effect and viscous friction.

Fig. 5. Stribeck curve: the circles represent experimental data and the solid line indicates the static part of the LuGre model

Table 1. Estimated Static Parameters of the LuGre Model

Parameter	Axis Moving East to West	Axis Moving West to East	Unit
σ_2	118.5822×10^2	123.6802×10^2	[N-m sec/radians]
T_C	200.5827	223.9246	[N-m]
T_S	1541.7090	1289.8430	[N-m]
v_S	0.00069	0.00084	[radians/sec]

3.4 Estimation of Dynamic Friction Parameters

Encoders mounted on the telescope can capture the presliding displacements of the order of 0.5×10^{-6} *radians*. When the driving torque is less than the breaking torque, very small displacements due to junctional deformations at the surface interface take place. In the bristle model, this is analogous to the bending of bristles without slipping. Therefore, the relative velocity ω will be same as $\frac{d\theta_z}{dt}$ and θ_z will change to a new steady state value. The system dynamics is given by:

$$u = J_L \ddot{\theta}_z + T_f \qquad (10)$$

where J_L is net reflected inertia of the telescope at the output shaft and u is the driving torque. T_f is the torque due to friction and is given by

$$T_f = \sigma_0 . \theta_z + (\sigma_1 + \sigma_2) . \dot{\theta}_z \qquad (11)$$

From equations 10 and 11 transfer function between θ_z and u can be obtained as:

$$\frac{\theta_z(s)}{u(s)} = \frac{1}{Js^2 + (\sigma_1 + \sigma_2)s + \sigma_0} \qquad (12)$$

This forms a second order LTI system about the equilibrium point for small displacements. The response of
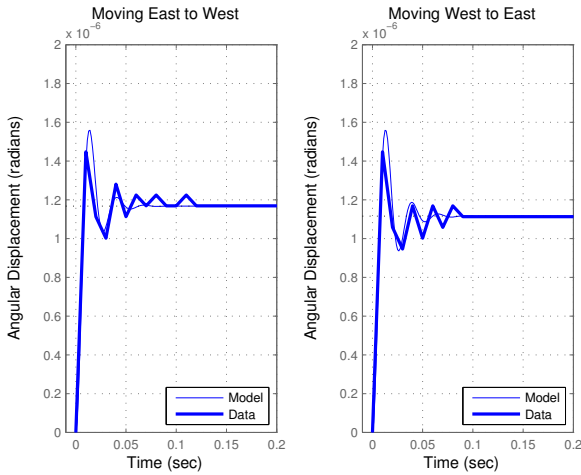
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Fig. 6. Thick line: the step response of the system; Thin line: the step response of the approximated second order model

the system to a step signal of small amplitude less than breaking torque is shown in Fig. 6.

The response is approximately similar to the step response of a second order system given by

$$\frac{\theta_z(s)}{u(s)} = \frac{K}{s^2 + 2\zeta\omega_n + \omega_n{}^2} \qquad (13)$$

Comparing the equations (12) and (13), we can write σ_1 and σ_0 as

$$\sigma_0 = \omega_n{}^2 J \qquad (14)$$
$$\sigma_1 = 2\zeta\omega_n J - \sigma_2 \qquad (15)$$

The parameters ζ and ω_n are easily estimated from the step response of the system and the system moment of inertia is already known. The dynamic parameters σ_0 and σ_1 obtained from these equations are shown in Table 2. Since the system is compared with a linear, mass-spring-damper governed by a second order differential equation, only a reasonable estimate of the parameters is obtained. To get more accurate estimates the nonlinearities need to be considered in the model.

Table 2. Estimated Dynamic Parameters of the LuGre Model

Parameter	Axis Moving East to West	Axis Moving West to East	Unit
σ_0	188.4800×10^6	158.0188×10^6	[N-m/radians]
σ_1	420.6185×10^3	385.1324×10^3	[N-m sec/radians]

To observe the hysteresis in the presliding regime the input torque is ramped up and down slowly with peak levels below the breaking torque. It is seen that the presliding displacements respond with a lag (Fig. 7).

4. CONCLUSIONS

The static friction parameters of the R.A. axis of the Schmidt telescope were determined using nonlinear curve fitting techniques in Matlab. Dynamics at presliding displacements can be described using second order LTI system and hence standard methods can be used for identifying the dynamic friction parameters. Experiments will be
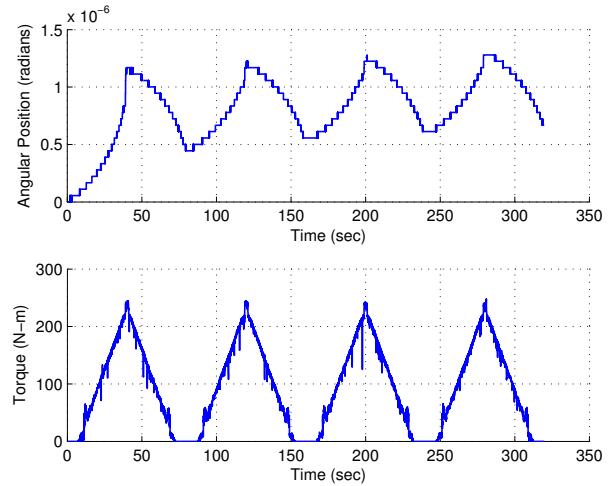

Fig. 7. Top: the system response; Bottom: the ramp input

performed on the telescope in a more elaborate manner for validating the LuGre model by determining other behavior of friction like repeatability, position dependencies, effect of dwell time, hysteresis etc. Successful estimation of friction model will motivate us to explore the compensation techniques to improve the overall telescope performance. After the procedure is completed, we hope to explore nonlinear control procedures for both tracking and regulation.

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